Number Systems 2, Binary	start time:

1. At the most fundamental level, computers use a binary number system.
The prefix **bi-** means two; a **binary** number system is one that has base = 2.

a.	How many symbols are needed to represent quantities in a binary system, given that base = 2?	
b.	What are the decimal values of the four smallest "places" in the binary number system? (For example, the first "place" would be $2^0 = 1$, which would be the "ones".)	

MODEL 1. Anatomy of a binary number

	0 ↑ 2^7 place ↓	**1** ↑ 2^6 place ↓	**0** ↑ 2^5 place ↓	**1** ↑ 2^4 place ↓	**1** ↑ 2^3 place ↓	**0** ↑ 2^2 place ↓	**0** ↑ 2^1 place ↓	**1** ↑ 2^0 place ↓
quantity	0	1	0	1	1	0	0	1
place	128	64	32	16	8	4	2	1
decimal value = quantity * place	0	64	0	16	8	0	0	1

2. Use Model 1 (above) to answer the following questions:

a.	Based on Model 1, what two symbols are used to represent quantities in a binary system?	
b.	What is the total value (in decimal) of the binary number 1011001?	
c.	In binary, all odd numbers have something in common. What is it?	
d.	In binary, all even numbers have something in common. What is it?	
e.	What are the decimal values of the next two places to the left of (greater than) the 128 place?	

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3. Convert each binary numbers to hexadecimal and decimal and complete the table.

	Binary	**Hexadecimal**	**Decimal**
a.	1010		
b.	11010		
c.	10011010		
d.	10101001		

INFORMATION

To complete the table above, many people first convert the binary number to decimal, and then convert that decimal value to hexadecimal. For example, 11010 (binary) is 16+8+0+2+0 = 26 (decimal), which is 1A (1 x 16 + 10 x 1) in hexadecimal. However, there is a simpler way to convert between binary and hexadecimal, based on the fact that $16 = 2^4$, so that each group of four digits in binary corresponds to one digit in hexadecimal.

For example, we can treat the binary number 1011001 as two "units" of four digits by adding zeroes to the left as needed:

0101 1001

We can then convert each of those into hexadecimal:

0101 = 5 1001 = 9

Thus, 1011001 converted to hexadecimal is 59.
This is equal to (5 x 16) + (9 x 1) = 80 + 9 = 89 (in decimal) which is the value that we obtained for 1011001 in Model 1 and Question 4.

4. To confirm that the "simpler way" described above is correct, use this shortcut to convert each binary number to hexadecimal, and then compare to your answers above.

	Binary	**Hexadecimal**
a.	1010	
b.	10011010	
c.	10101001	

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5. This approach can be applied to binary numbers of any length. In some cases, converting from binary to hexadecimal and then to decimal can be simpler than converting to decimal directly.
For example, we will convert **101100011000** from binary to hexadecimal:

a.	Split the binary number into three units each with four digits:			
b.	Convert each unit to a single hexadecimal value:			
c.	Enter the place value for each hexadecimal place.			
d.	Multiply b x c.			
e.	Add the 3 columns for the final answer.			

6. Show how you can use this approach to convert 2584 hexadecimal to decimal.

7. You should now understand how number systems work and how to convert between decimal and binary and hexadecimal. As a team, choose a number that is not 2, 10 or 16 to be a base and create a number system for that base.
   a. Complete the table(s) on the next page with your own number system.
   b. Have each team member convert at least one number.
   c. Record your results and observations below.

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	↑	↑	↑	↑	↑	↑	↑	↑
	place	place	place	place	place	place	place	place
	↓	↓	↓	↓	↓	↓	↓	↓
quantity								
place								
value=Q*P								

	↑	↑	↑	↑	↑	↑	↑	↑
	place	place	place	place	place	place	place	place
	↓	↓	↓	↓	↓	↓	↓	↓
quantity								
place								
value=Q*P								

	↑	↑	↑	↑	↑	↑	↑	↑
	place	place	place	place	place	place	place	place
	↓	↓	↓	↓	↓	↓	↓	↓
quantity								
place								
value=Q*P								

	↑	↑	↑	↑	↑	↑	↑	↑
	place	place	place	place	place	place	place	place
	↓	↓	↓	↓	↓	↓	↓	↓
quantity								
place								
value=Q*P								

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