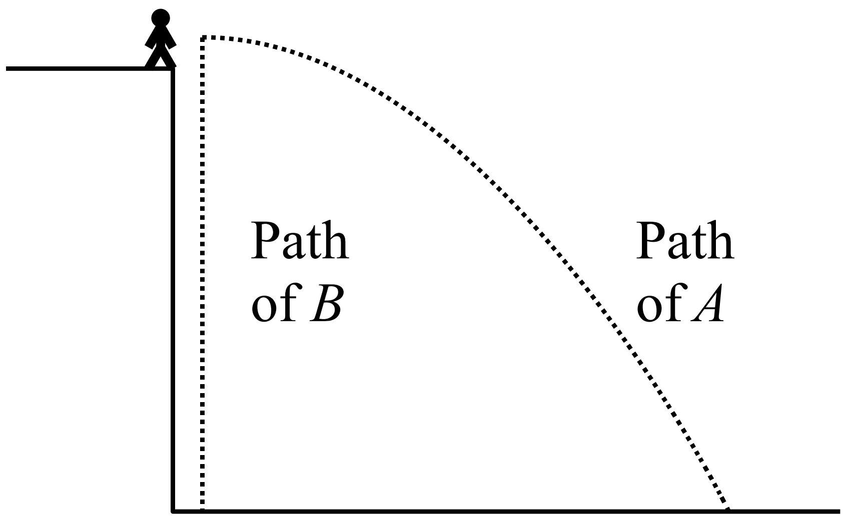
Kinematics in 2D and 3D

Another message to you from a former student:

I can almost guarantee you that using (the BSC approach) for learning physics will seem completely unnatural at first, and, personally, I found myself still thinking the same thing towards the end of second semester as well. Yes, I was one of the resisters of BSC's ways. However, let me say that if I could do it again, I would. When it came to studying for the MCAT, I realized upon opening my first Kaplan physics review book that I would have the easiest time reviewing physics more than anything else. By learning physics the way I did (at the most fundamental level) I found it easier to recall key physics concepts and apply them to a variety of complex problems and circumstances. All of those "silly" overhead problems…trust me, they help you more than you'll ever know.

A person standing at the edge of a cliff throws rock *A* horizontally off the cliff and at the same time drops rock *B* straight down into the canyon below. Which rock should hit the flat canyon floor first?



1. Rock *A*
2. Rock *B*
3. Both hit at the same time
4. It depends on the initial horizontal velocity of *A*.

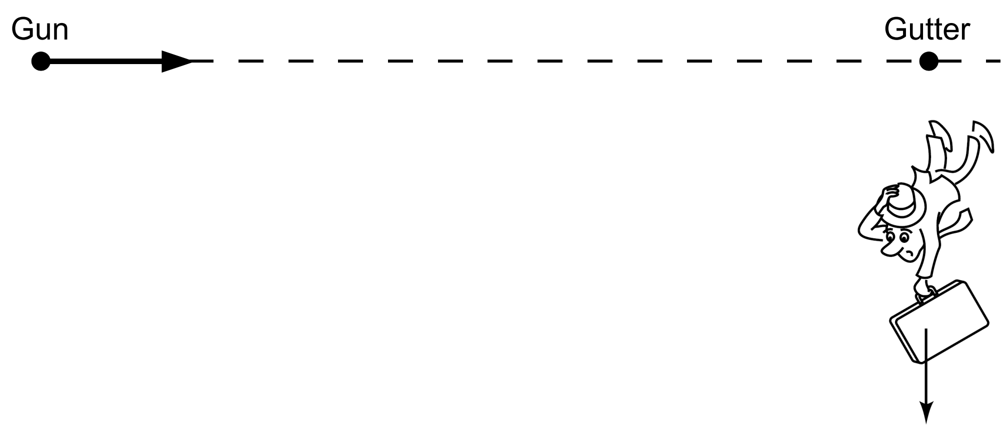
ANS: **3**—Both will hit at the same time.

In free flight, the horizontal and vertical motions of an object are independent of each other. The kinematics of horizontal and vertical motion are independent.

In particular, this means that the time it takes the stones to hit the ground is determined only by kinematic quantities in the vertical direction: the height they fall, their initial vertical components of velocity, and their vertical acceleration. These quantities are the same for both rocks.

Recall the class demo that that was exactly equivalent to this. I launched one ball sideways at the same instant that I dropped the other one straight down. To within the limits of your hearing, it sounded like both balls hit the floor at the same instant.

A dangerous criminal hangs from the gutter of a building, and a sniper on the same floor of an adjacent building has his rifle aimed exactly at the criminal. The target is well within the gun's range, but the *instant the gun is fired*, the criminal lets go and drops to the ground. What happens?



1. The bullet hits the criminal regardless of its initial speed.
2. The bullet hits the criminal only if its initial speed is large enough.
3. The bullet misses the criminal.

ANS: **1**—The bullet hits the criminal regardless of its initial speed.

Both the criminal and the bullet will fall under the influence of gravity, so in the time it takes the bullet to reach the criminal’s horizontal position, both will have fallen the same distance. Strictly speaking, that means that if the criminal had not let go, the bullet would have hit a point below where the sniper had aimed.

As long as the initial velocity of the bullet is horizontal, and the criminal lets go at exactly the same instant the bullet leaves the gun, the bullet will hit regardless of how fast it is moving. The “sniper” could have thrown a baseball, and the falling criminal could have caught it, as long as the baseball traverses the horizontal distance before the criminal hits the ground. Once the criminal hits the ground the problem is no longer governed by free-flight kinematics.

A research biologist needs to sedate a particularly troublesome monkey that has climbed up a tree. Having worked with this monkey before, she knows that the monkey will let go of the tree and fall at the instant she fires her dart gun. Where should she aim the dart gun to ensure she will hit the monkey before it lands and scampers off?

1. Aim above the monkey
2. Aim directly at the monkey
3. Aim somewhere below the monkey (i.e. “lead” it)
4. Aim horizontally
5. Need more information.

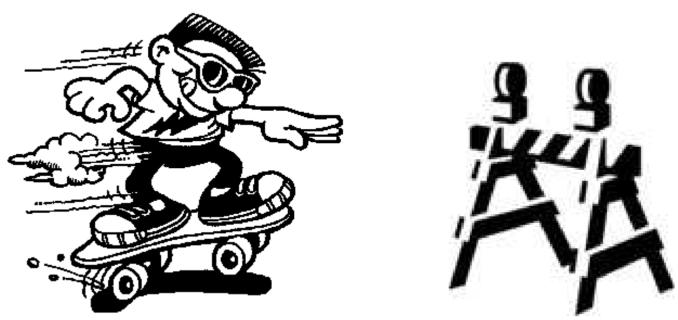
ANS: **2**—She should aim directly at the monkey.

This is very much like the falling criminal problem, but the initial conditions are different. It may seem that the initial vertical component of the dart's velocity changes things. After all, the monkey will have zero initial vertical velocity. Does this difference affect the outcome?

No, it does not. Imagine, for a moment, that there were no gravity. The monkey will let go, but it will not fall. The dart will fly in a straight line at constant velocity, because there is no acceleration in the problem. Therefore, in order to hit the monkey, she would have to aim directly at it. The initial horizontal and vertical components of the dart's velocity are precisely the values necessary to get the dart to the monkey's position at the top of the tree.

Now let's turn gravity on. The kinematic equations for both the monkey and the dart will have an additional vertical component due to gravitational acceleration, $\frac{1}{2}a_g t^2$. This term is the same for both objects, so in the time the dart is in the air, both the monkey and the dart will "drop" equal distances due to gravitational acceleration. In other words, the dart will fall below its non-gravitational straight-line path by exactly the same distance that the monkey will fall from its starting point.

A skater is cruising down his favorite flat road when he sees a barrier across his path. He can easily jump high enough to clear the barrier, and his board will roll underneath it just fine. As he approaches, he suddenly realizes that he’s not sure *how* he should jump so that he is sure to land on his skateboard after he clears the barrier. What’s his best strategy?



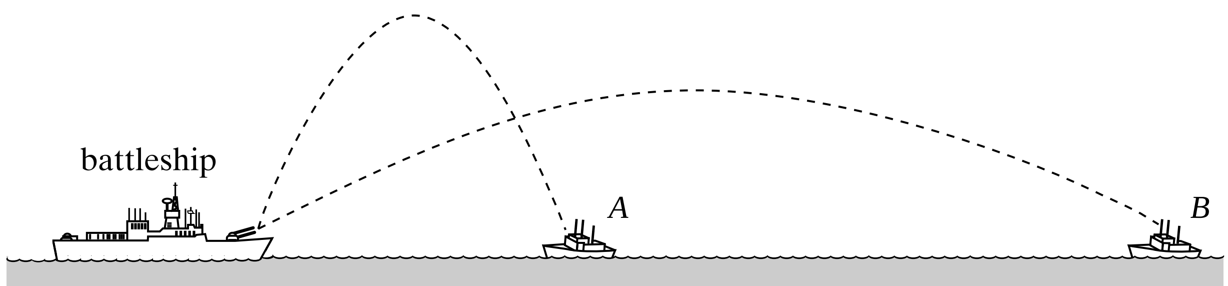
1. Jump straight up from the board
2. Jump up and forward.
3. Jump up and backward.
4. It depends on how fast he is going.

ANS: **1**—The skater should jump straight up from the board.

As long as the skater jumps straight up, neither the board's velocity, nor the skater's horizontal component of velocity will change. Remember, he does not need the board to keep him moving forward. If he gives himself a vertical velocity, his horizontal motion will remain unchanged. He will still move forward at the same speed as the skateboard, passing over the barrier at the same time the board passes under.

Recall the class demo I ran with the cart that fired the ball straight up. When the cart and ball were at rest, the ball would land back in the launcher cup. When the cart moved at constant velocity, the ball still landed in the cup because the cart and ball maintained the same horizontal velocity throughout the trip.

A battleship simultaneously fires two shells at enemy ships. If the shells follow the parabolic trajectories shown, which ship gets hit first?



1. *A*
2. *B*
3. Both at the same time
4. Need more information

ANS: **3**—Ship *B* will be hit first.

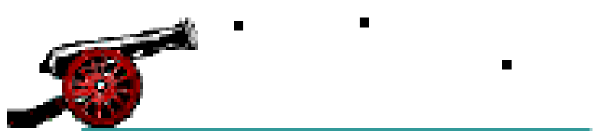
You have all the information you need to answer this question right in the picture. You don't know the initial firing angles, but they're clearly different for the two shells. You don't know the initial velocities of each shell, either, but they're almost certainly different. These unknown quantities don't matter anyway!

What does matter is the height of each shell's trajectory. The two paths are symmetric. In both cases, the shells start and end vertically at ship level. Each shell rises the same distance that it later falls. The total flight time for each shell, therefore, it twice the time it takes the shell to fall from the highest point of its trajectory to hit the target.

The time of fall is governed by the vertical kinematics. Starting at the highest points of both trajectories, both shells have zero vertical velocity. They have equal vertical (gravitational) accelerations. The difference is the fall height. Shell *B* falls a shorter distance, so it takes less time to hit its target. That means the the total flight time for *B* is less than for *A*.

Warmup Question

To attain the maximum range, a projectile has to be launched at 45 degrees above the horizontal if the landing spot and the launch spot are at the same height (neglecting air resistance effects.) Explain why raising the angle above 45° reduces the range. Then explain why lowering the angle below 45° also reduces the range.



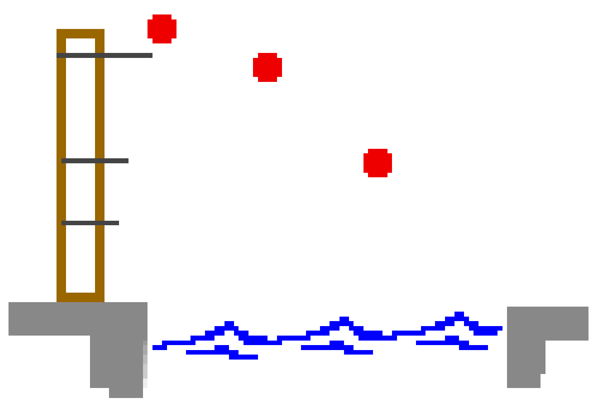
ANS: We are given that the 45° angle gives the maximum range. Our job is to figure out why other angles give a lower range.

Raising the angle will give a much greater vertical component of velocity, so the projectile will reach a much greater height and will spend a longer time in the air. However, the horizontal component of velocity is much lower than at a 45° angle, so the horizontal distance traveled might not be as great even though more time is spent in the air. The extreme case is for the canon fired at a 90° angle, where the ball rises straight up and falls back to the same spot. In that case the range is zero. Apparently the competing factors of longer flight time and smaller horizontal velocity give a maximum range for a 45° shot.

Contrariwise, at a lower angle the horizontal component of velocity is much greater than at 45° . However, because of the smaller vertical component of velocity, the projectile will not rise very high and will return to the ground in a very short time. The extreme case is for the canon fired at 0° . If the canon barrel is on the ground (not on wheels, as shown) the ball will hit the ground immediately after leaving the gun, giving a range of zero. Apparently the competing factors of shorter flight time and larger horizontal velocity give a maximum range for a 45° shot.

Warmup Question

The Olympic diving tower is 10 meters high, and the top platform is about 3 meters long. A good runner could get into trouble by taking a running dive and overshooting the length of the diving pool. Make a reasonable assumption about a good runner’s maximum speed and estimate the minimum length for a “safe” diving pool.



ANS: We need to estimate the initial horizontal velocity of the diver, whom we take to be a "good" runner. The fastest sprinters can run at approximately 10 m/s. Our diver probably cannot run as fast, so by taking this nice round value we will safely over-estimate the length of the pool.

Second, we need the fall time to determine how far, horizontally, the diver will land from the board. To do this, we need to know how long the diver is in the air, or the fall time. The diver has no initial vertical component of velocity, so we can use the kinematic equation $\Delta y = \frac{1}{2} a_g t^2 \;\rightarrow\; t = \sqrt{2h/a_g}$. Using $h = 10\,\mathrm{m}$ and $a_g \approx 10\,\mathrm{m/s^2}$, we get $t \approx 1.4\,\mathrm{s}$. Using $\Delta x = v_x \Delta t$, we get $\Delta x \approx 14\,\mathrm{m}$ beyond the end of the diving board, or 17 m long in total. This corresponds to around 55 ft. Again, this is probably a serious overestimate.

Warmup Question

A football is kicked at ground level in such a way that the horizontal component of its initial velocity is 7 m/s, while the vertical component is 10 m/s. Roughly what will the speed of the football be when it returns to the ground (ignoring air resistance)?

1. 0 m/s
2. 7 m/s
3. 10 m/s
4. 12 m/s.

ANS: **4**—Around 12 m/s.

This is a projectile (free-flight) problem, so we know that the horizontal component of velocity will be constant at 7 m/s. To find the vertical component, you should recognize that because the ball lands at the same height it started from, the upward and downward motions are just reverses of each other. Therefore, the vertical component of velocity when it returns to the ground will be 10 m/s downward. So the total velocity right before hitting the ground is (defining up to be the $+\hat{\boldsymbol{\jmath}}$ direction):

$$\vec{\boldsymbol{v}} = (+7\,\hat{\boldsymbol{\imath}} - 10\,\hat{\boldsymbol{\jmath}})\,\mathrm{m/s}\,.$$

This gives a speed

$$v = \sqrt{(+7\,\mathrm{m/s})^2 + (-10\,\mathrm{m/s})^2} = 12.2\,\mathrm{m/s}\,.$$

Even if you didn't know to use the Pythagorean theorem, you could have guessed that 12 m/s was the best answer because with velocity components of 7 m/s and 10 m/s, the speed (magnitude of velocity) will be greater than either of these.