

Old fashion bicycles have fixed-gear drives similar to what is shown below. Rather than turning the wheels themselves, riders turn pedals attached to a toothed disk upon which a chain rides without slipping. The same chain rides on a second, smaller toothed disk on the back tire. Discuss how the chain relates the rotational motion of the two disks to each other. Explain your logic.



Right conclusion, incomplete logic

For this example, let's say that the bike is being ridden in a forward motion. The angular displacement of both of these disks will be counterclockwise. **The two disks should have different angular velocity since the disks are different. The number of rotations of the smaller disk will be higher in comparison to the larger disk.**

Right conclusion, better logic, still not the whole story

The chain connects the two discs as I would imagine a chain connects two gears. The radius of a circle does not matter in terms of angular displacement, so the angular displacement of each toothed disk rotating in a full circle is 2π radians. However, **because the radius of one disk is smaller than the other disk, the time it takes to complete a full circle is different.** Therefore, the angular speed of each disk is different from one another. In this scenario, the chain transfers the rotation to the first gear caused by pedaling to rotation of the second gear, which causes the wheel to turn. When the larger gear is displaced by a certain amount, the second gear must rotate much faster to match the displacement of the first gear.

Key constraint: Tangential speeds (!) must be the same

The chains are in rotational motion because they travel a path that rotates in a oval path around two disks. **This chain has the same linear velocity at every point, even on both disks.**

Putting it all together

The speed at which the chain is moving is the same as the speed at the outside of the circles it is attached to. If you follow a single point on the chain, and measure its speed, with constant peddling the distance it travels along the chain during a specific amount of time should be constant. If the distance it is traveling per unit of time is constant, that means that **it takes longer to travel around the circumference of the larger disc** than the smaller disc. Because it takes longer for the point to make a full rotation on the larger disc, it is safe to say **the larger disc is spinning slower than the smaller disc.**

Estimate the magnitude of the tangential velocity of an object in Birmingham due to the rotation of the Earth. Describe your logic. (If you promise not to get too hung up on being precise, the distance from the Earth's pole to the equator is 10,000 km, which originally defined the kilometer.)

Good logic even though using miles until a calculator raises its ugly head

Since the object is (I'm assuming) not moving, the tangential velocity of the object is really the tangential of Birmingham. **The earth takes 24 hours for a fully rotation and its circumference is 25,000 miles.** 25,000/24 is **basically 1,000 miles of movement per hour.** The stationary object in Birmingham has a tangential velocity of 1000 miles per hour or **447.028 meters per second.**

Good answer with consideration of latitude, but too precise at end

According to my globe, **Alabama is about 36 degrees above the equator.** If the equators radius is 10,000 km, Birmingham's radius is $\cos(36) * 10,000$ km or **about 8000 km from the axis of rotation.** Therefore, the circumference is $8000 \text{ km} * \pi$ or **about 25,000 km / 1 rotation.** Since the earth rotates 1 time per day, the rps would be 1 rotation / 3600 sec * 24 hrs or about 1 rotation / 85000 seconds. As such, the tangential velocity would be 1 rotation / 85,000 seconds * 25,000 km / 1 rotation or **25,000 km/ 85000 sec or 0.294 km/sec or 294 m/s.**

Finally, a sensible estimation with all work shown!

Tangential velocity would be calculated by the displacement in terms of the Earth's spherical surface over time. The Earth completes one rotation each day:

$(1\text{day})(24\text{hr}/1\text{day})(60 \text{ min}/1\text{hr})(60 \text{ s}/1 \text{ min})=24*3600 \text{ s about } 8*10^4 \text{ s.}$

The circumference of the Earth is $2*\pi*\text{radius}$ which is $2*\pi*10,000\text{km}$ or **$(2*10^4)\pi \text{ km.}$**

Therefore the tangential velocity would be **$((2*10^4)\pi \text{ km})/(8*10^4\text{s})=(\pi/4)\text{km/s}$**

The angular velocity of the Earth orbiting around the Sun is approximately

- a. 365 radians/day
- b. 0.017 radians/day
- c. 12 radians/yr
- d. 1.0 radians/yr