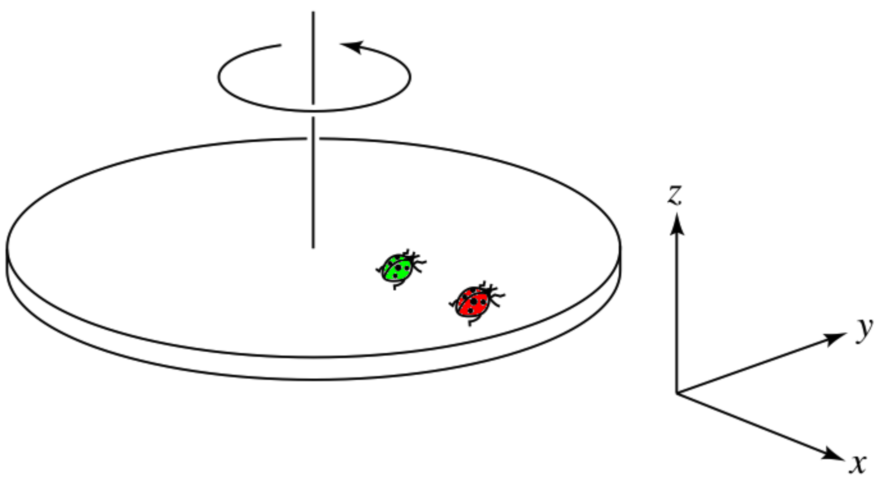
Rotational Kinematics

Think of homework as a challenging puzzle, like Sudoku. When you’re at the point that you really understand it, you don’t need the solution for confirmation.

A ladybug sits at the outer edge of a merry-go-round, and a gentleman bug sits halfway between her and the axis of rotation. The merry-go-round makes a complete revolution once each second. The gentleman bug's angular speed is

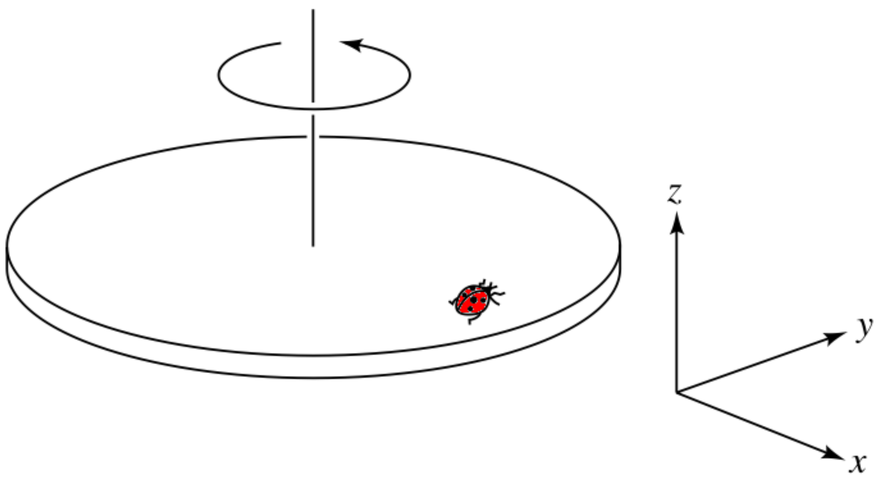


1. half the ladybug's.
2. the same as the ladybug's.
3. twice the ladybug's.
4. impossible to determine

ANS: **2**—The gentleman bug's angular speed is the same as the ladybug's.

This is an example of rigid-body rotation, where all points on the body rotate together. Both make one complete circle ($\Delta\theta = 2\pi$ rad) in 1 s, so both have an angular sped of 2π rad/s.

A ladybug sits at the outer edge of a merry-go-round, that is turning and slowing down. At the instant shown in the figure, the *tangential* component of the ladybug's (linear) acceleration is

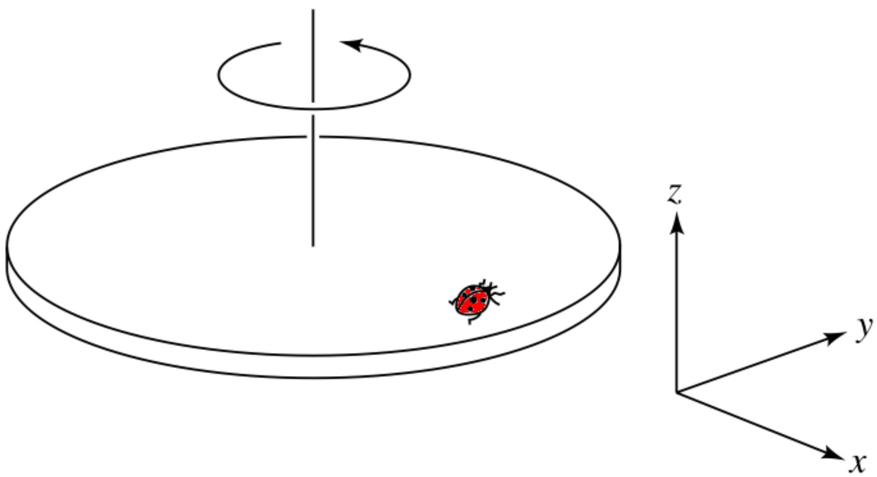


1. in the $+x$ direction.
2. in the $-x$ direction.
3. in the $+y$ direction.
4. in the $-y$ direction.
5. in the $+z$ direction.
6. in the $-z$ direction.
7. zero

ANS: **4**—The tangential component of acceleration is in the $-y$ direction.

The ladybug's (translational) velocity points in the $+y$ direction. The tangential acceleration, which is related to changes in speed, points opposite her velocity vector because she is slowing down. Therefore, $\vec{\boldsymbol{a}}_t$ points in the $-y$ direction.

A ladybug sits at the outer edge of a merry-go-round, that is turning and slowing down. At the instant shown in the figure, the *radial* component of the ladybug's (linear) acceleration is

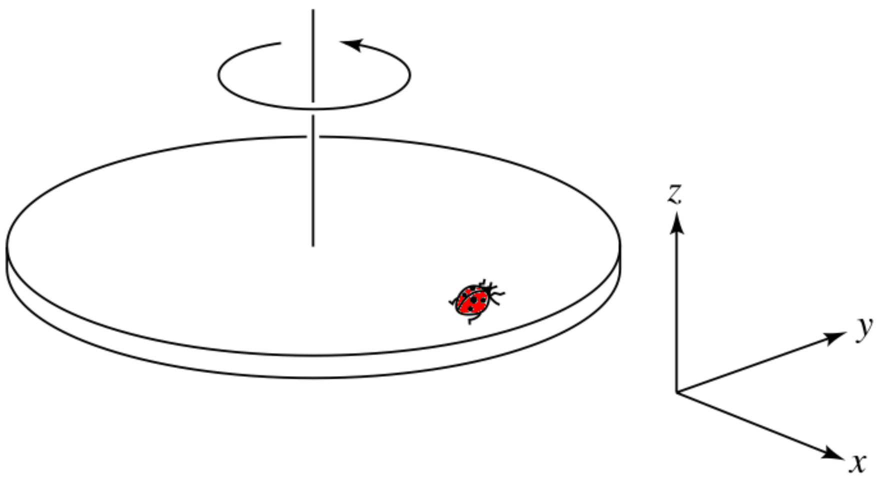


1. in the $+x$ direction.
2. in the $-x$ direction.
3. in the $+y$ direction.
4. in the $-y$ direction.
5. in the $+z$ direction.
6. in the $-z$ direction.
7. zero

ANS: **2**—The radial acceleration points in the $-x$ direction.

Her radial (direction-changing) acceleration points toward the center of the circular path her motion traces out. This is the $-x$ direction on the diagram.

A ladybug sits at the outer edge of a merry-go-round, that is turning and slowing down. At the instant shown in the figure, the vector expressing her *angular velocity* is

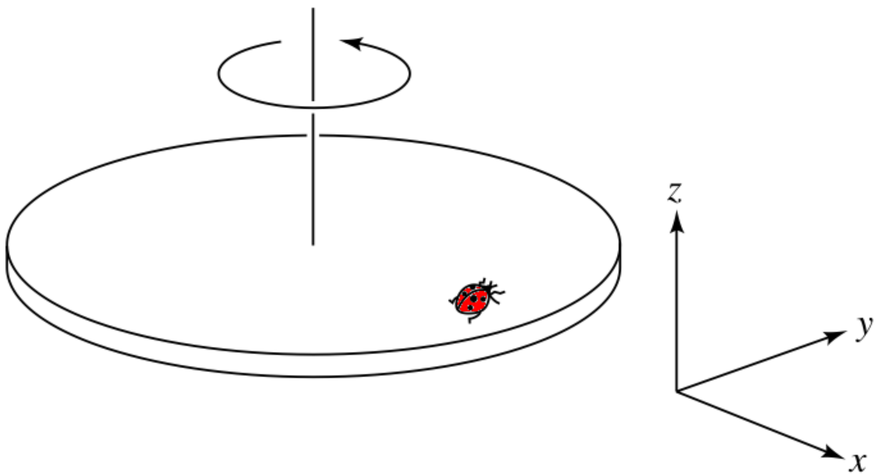


1. in the $+x$ direction.
2. in the $-x$ direction.
3. in the $+y$ direction.
4. in the $-y$ direction.
5. in the $+z$ direction.
6. in the $-z$ direction.
7. zero

ANS: **5**—The angular velocity vector points in the $+z$ direction.

To find the direction of angular velocity, you need to use a right-hand rule. Curl your fingers of your right hand in the direction of rotation. Your thumb will point in the direction of angular velocity.

A ladybug sits at the outer edge of a merry-go-round, that is turning and slowing down. At the instant shown in the figure, the vector expressing her *angular acceleration* is



1. in the $+x$ direction.
2. in the $-x$ direction.
3. in the $+y$ direction.
4. in the $-y$ direction.
5. in the $+z$ direction.
6. in the $-z$ direction.
7. zero

ANS: **6**—The angular acceleration vector points in the $-z$ direction.

Just like acceleration is a measure of change in velocity, angular acceleration is a measure of change in angular velocity. In this case the angular velocity points in the $+z$ direction and is decreasing. The angular acceleration, therefore, will point opposite the direction of angular velocity, or the $-z$ direction.

Warmup Question

Old fashion bicycles have fixed-gear drives similar to what is shown below. Rather than turning the wheels themselves, riders turn pedals attached to a toothed disk upon which a chain rides without slipping. The same chain rides on a second, smaller toothed disk on the back tire. Discuss how the chain relates the rotational motion of the two disks to each other. Explain your logic.

ANS: Every link in the chain moves with the same speed. (If this were not so, the chain would be stretching or compressing.) As the chain links roll over the toothed gears, the tangential speed of each gear tooth will be the same as the speed of each chain link. Therefore, the chain guarantees that each gear has the same tangential speed for its teeth. The smaller gear disk will have a greater angular speed, in proportion to the ratio of the gears' radii.

Warmup Question

Estimate the magnitude of the tangential velocity of an object in Birmingham due to the rotation of the Earth. Describe your logic. (If you promise not to get too hung up on details, the distance from the Earth's pole to the equator is 10 000 km, which originally defined the kilometer.)

ANS: The hint points out that Earth's circumference is essentially 40 000 km, so a point on the equator has a tangential speed of 40 000 km/day. Birmingham is well north of the equator, so the tangential velocity will not be as great. A point with $60°$ latitude will have half the equatorial velocity, because it's distance from Earth's axis will be half as great ($\cos 60° = 1/2$). At Birmingham's latitude (slightly greater than $30°$, the distance from Earth's axis will be slightly more than 80% of the equatorial radius, giving a circumference of greater than 32 000 km and a speed in excess of 32 000 km/day, or over 1300 km/hr.

Warmup Question

The angular speed of Earth orbiting around the Sun is approximately

1. 365 radians/day
2. 0.017 radians/day
3. 12 radians/yr
4. 1.0 radians/yr

ANS: **2**—The angular speed of Earth's orbit is approximately 0.017 radians/day.

Earth completes one orbit (2π radians) in one year, giving it an orbital angular speed of 6.28 radians/year). Therefore, neither choice 3 nor choice 4 can be correct. Choice 1 is also certainly not correct. In reality, Earth's angular speed is $(2\pi \text{ radians/year})(1 \text{ year}/365 \text{ days}) = 0.017$ radians/day.