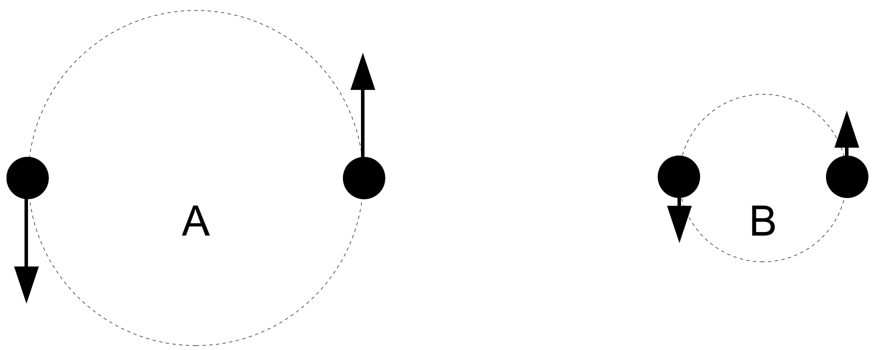
Rotational Inertia

If you can’t explain something in words to someone else, you don’t really understand it yourself.

Two point particles, each of mass m, undergo circular motion about their center of mass as in the pictures below. The masses in system A are separated by twice the distance between the masses in system B. Both systems make one complete revolution every second. Compare the total kinetic energies of the two systems.

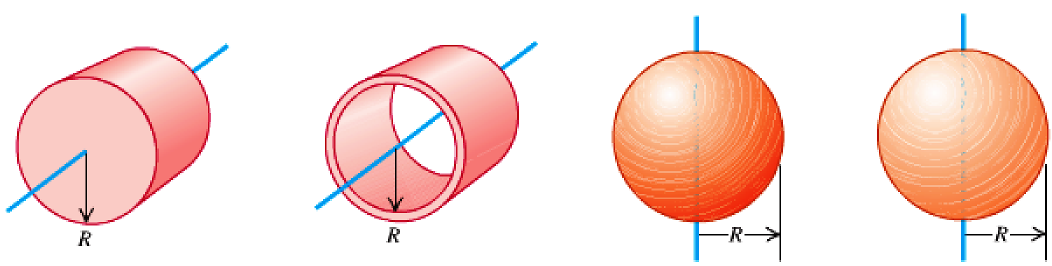


1. $K_A = K_B = 0$ because the centers of mass don't move.
2. $K_A = K_B > 0$ because the masses rotate at the same rate in both cases.
3. $K_A > K_B$ because the masses in system A move faster than the masses in system B.

ANS: **3**—$K_A > K_B$

The masses circle the centers with the same angular velocity. However, the masses in system A are farther from the center and therefore have greater speeds. Therefore, they have greater kinetic energy.

The picture below shows four objects that have the same mass and radius. The objects are, in order from left to right: a uniform cylinder, a hollow cylinder (ring), a solid sphere, and a hollow spherical shell. Which of these would you expect to have the greatest rotational inertia ("moment of inertia") for rotations about the axes shown? ***Don't look up formulas in your book!!!!***

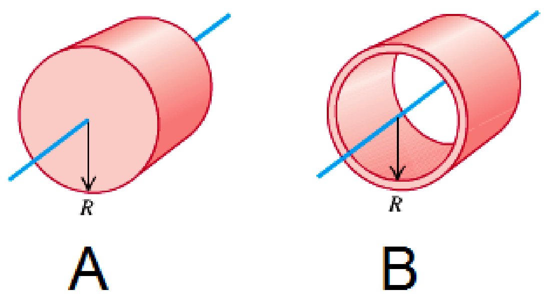


1. The solid cylinder
2. The hollow cylinder
3. The solid sphere
4. The hollow spherical shell
5. They should all be the same
6. More information needed

ANS: **2**—The hollow cylinder has the greatest rotational inertia.

Moment of inertia is greatest when all of the mass is far from the rotation axis. All of the objects have the same radius, *R*. For the hollow cylinder, all of the mass is located a distance *R* from the rotation axis. For every other object, some of the mass is closer to the axis of rotation. The hollow sphere has the next greatest rotational inertia, followed by the solid cylinder and the solid sphere.

Two cylinders of the same size and mass roll down an incline. Cylinder A is a uniform disk. Cylinder B has most of its weight concentrated at the rim, so it's basically a hoop. Which reaches the bottom of the incline first?



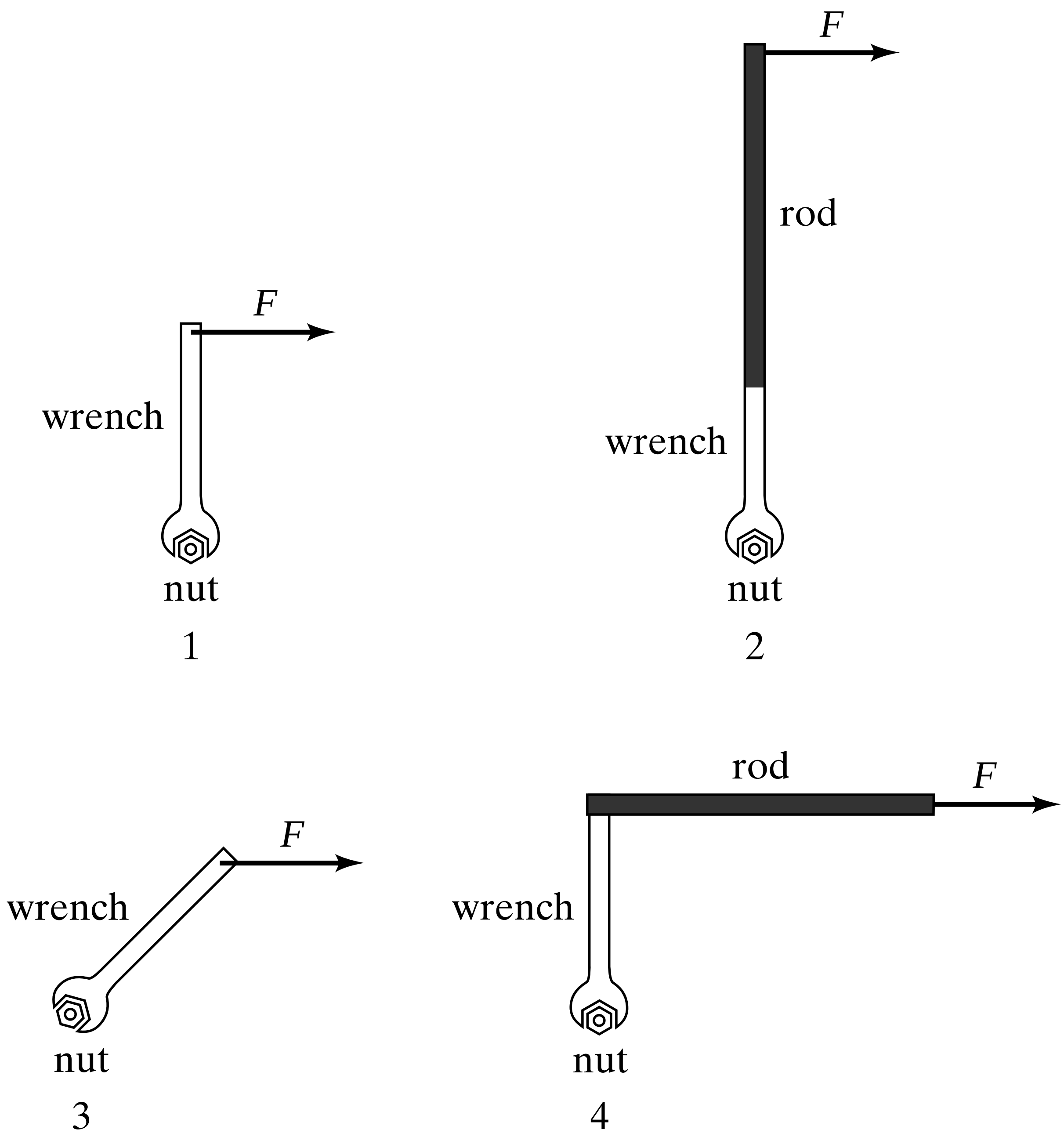
1. Cylinder A — the disk
2. Cylinder B — the hoop
3. Both reach the bottom at the same time.

ANS: **1**—The disk reaches the bottom first.

Like mass, rotational inertia is a measure of resistance to change in motion (in this case, rotational motion). The same forces act on the cylinders in the same way (same torques), so the one with the smallest rotational inertia should have the greatest angular acceleration.

It is also very useful to think of this in terms of energy. The cylinders have the same gravitational potential energy at the top, so they have the same total kinetic energy (translational plus rotational) at the bottom. Because both have the same radius, both have the same relation $v = r\omega$ between translational and rotational velocities. Because of its larger rotational inertia, a greater amount of the hoop's total kinetic energy will be associated with rotational energy for the same speed. For the same total kinetic energy, on the other hand, the hoop will not have a speed as great as the disk's.

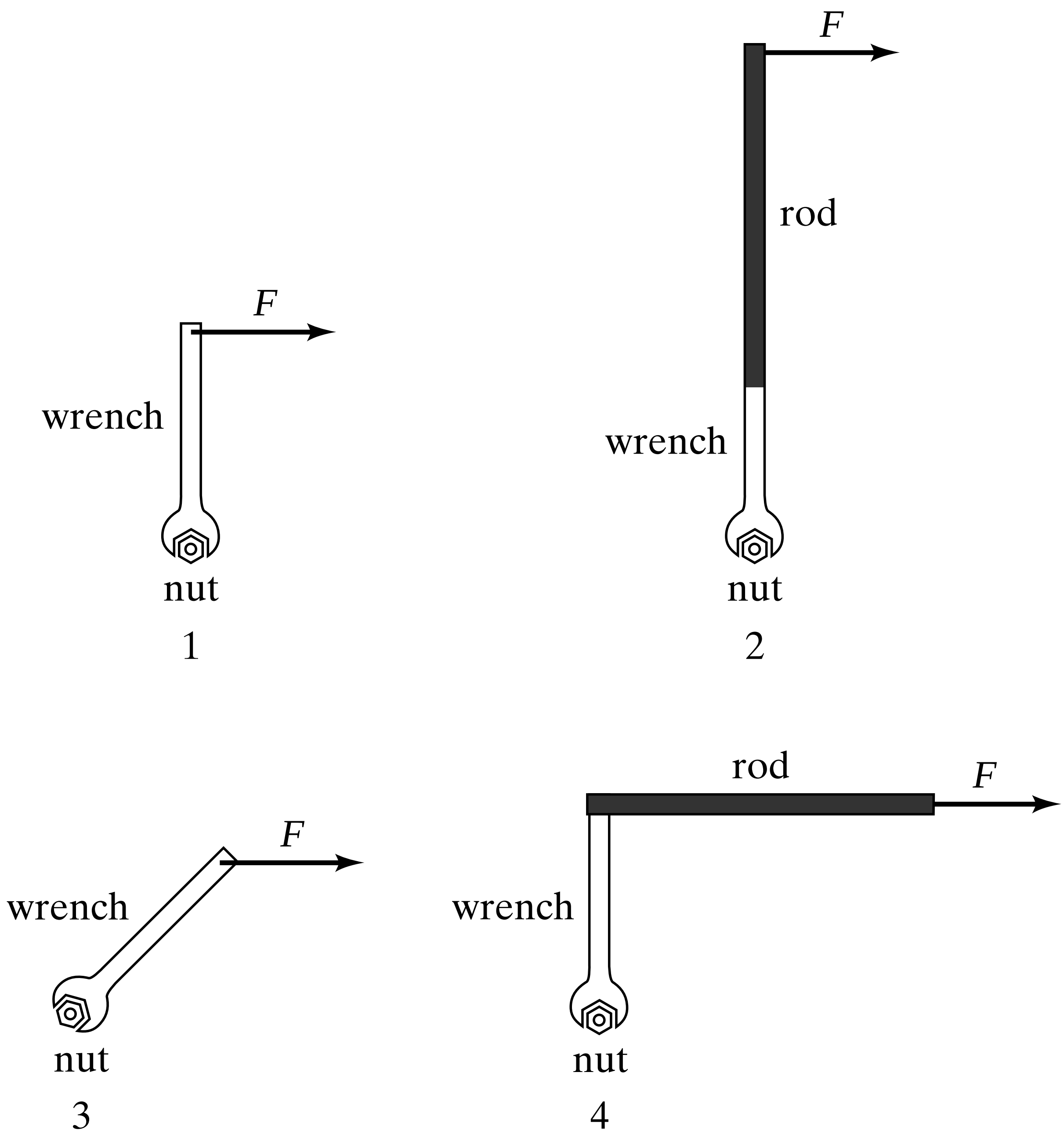
You are using a wrench and trying to loosen a rusty nut. Which of the arrangements shown is *most* effective in loosening the nut?



ANS: Arrangement **2** is most effective in loosening the nut.

The force is applied to a longer moment arm for this arrangement than for any other. This means that arrangement 2 has the greatest applied torque.

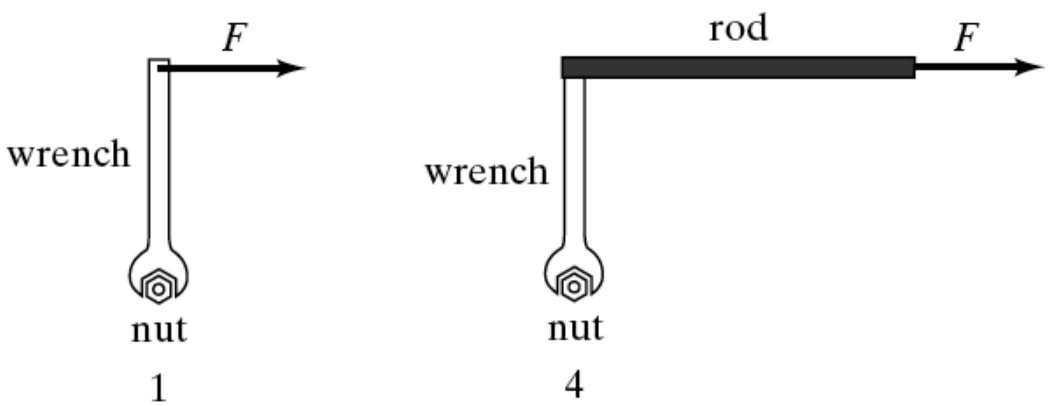
You are using a wrench and trying to loosen a rusty nut. Which of the arrangements shown is *least* effective in loosening the nut?



ANS: Arrangement **3** is least effective in loosening the nut.

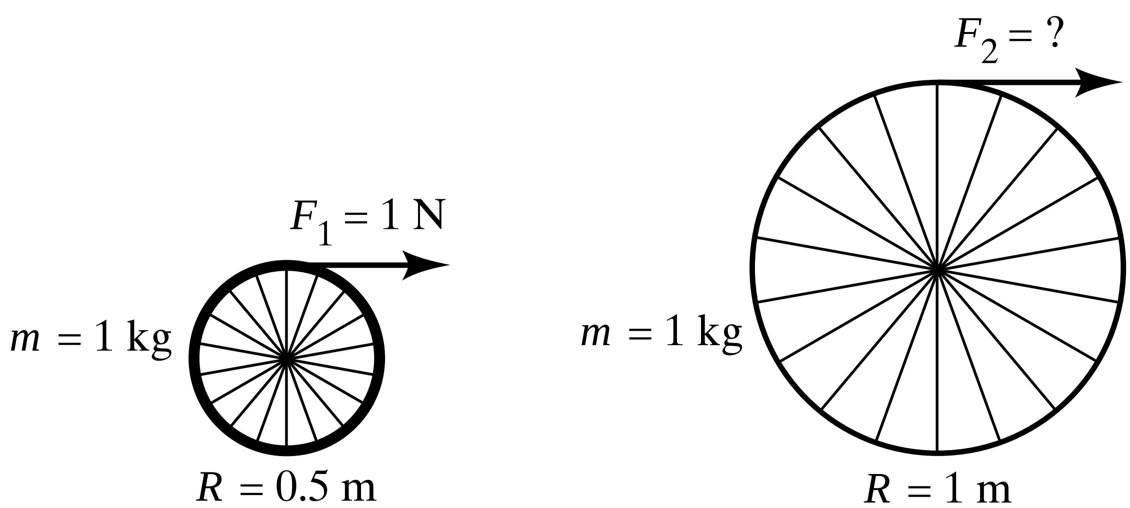
The force is applied to a shorter moment arm for this arrangement than for any other, so this arrangement has the smallest applied torque.

Finally, there are two remaining cases, numbers 1 and 4. Which of those two is most effective in loosening the nut?



1. 1
2. 4
3. They are equally effective

Two wheels with fixed hubs, each having a mass of 1kg, start from rest, and forces are applied as shown. Assume the hubs and spokes are massless, so that the rotational inertia is $I = mR^2$. How large must F_2 be to give the wheels the same angular accelerations?



1. $0.25\,\mathrm{N}$
2. $0.5\,\mathrm{N}$
3. $1\,\mathrm{N}$
4. $2\,\mathrm{N}$
5. $4\,\mathrm{N}$

ANS: **4**—$F_2 = 2\,\text{N}$.

The wheels have the same mass, but the larger wheel has twice the radius and therefore four times the rotational inertia of the smaller wheel. To get the same angular acceleration we would need to apply four times the torque to the larger wheel than to the smaller one. Torque is the product of the wheel's radius and the tangential force. Since the larger wheel has twice the radius, we only need to apply twice the tangential force to the larger wheel to get four times the torque applied to the smaller wheel.

Jack and Jill climb two identical hills (same height and slope) to fetch pails of water. Jack’s hill is icy, so when he falls he slides down the hill. Jill’s hill is grassy, so when she tumbles down her hill she rolls without slipping.

Who is moving faster at the bottom of the hill? (Consider only motion prior to any headgear-damaging impacts, of course.)

1. Jack
2. Jill
3. They’ll have the same (translational) speed
4. Need more information

ANS: **1**—Jack is faster at the bottom.

If they have the same mass, both have the same initial gravitational potential energy. Let's define the gravitational potential energy to be zero at the bottom of the hill. All of Jack's motion is translational, so at the bottom of the hill, all of his energy is translational kinetic. Jill's motion is both translational and rotational. At the bottom of the hill, her total kinetic energy will be the same as Jack's, but only part of it will be translational (and part will be rotational). Therefore, her translational speed will be less than Jack's. (Actually, because gravitational potential, translational kinetic, and rotational kinetic energies are all proportional to mass, the answer will be the same even if Jack and Jill have different masses. Mass divides out of the problem.)

Warmup Question

Balancing a broom on your palm with the heavier, straw end upward isn’t very hard. In contrast, trying to balance a broom with the straw end downward against your palm is very difficult (and this has nothing to do with the stiffness of the straw!) Discuss why this is so. (You are encouraged to try it out before answering.)

ANS: Having the heavy brush side highest gives the greatest rotational inertia (most mass away from the center of rotation) for rotations around your hand. If the broom starts to fall, it will fall more slowly (smaller angular acceleration) with the mass far from the hand compared to near the hand. Because of the slower fall, you will have more time to move your hand to correct the balance and keep the broom upright.

Warmup Question

Estimate your own moment of inertia. Explain your logic and don’t forget the units!

ANS: I cannot answer this question until I define around which axis I would rotate. For example, I may want to treat myself as a thin rod (Hah!) rotating around a horizontal axis at my feet. This would be relevant for problems involving me tripping and falling—something I have a great deal of experience with. In that case, my rotational inertial would be $I = \frac{1}{3}ML^2$. With $M = 100\,\mathrm{kg}$ and $L = 1.75\,\mathrm{m}$, my rotational inertia would be around $100\,\mathrm{kgm}^2$.

Suppose, on the other hand, that I am rotating around a vertical axis through the core of my body, with my hands at my sides. (Perhaps I am rolling out of bed onto the floor.) In that case it is better to model me as a uniform cylinder with moment of inertia $I = \frac{1}{2}MR^2$. With $M = 100\,\mathrm{kg}$ and $R = 0.25\,\mathrm{m}$, my rotational inertia would be around $3\,\mathrm{kgm}^2$. Clearly it is easier to spin people than to flip them end over end!

Warmup Question

Without looking back at the formulas (really!), which of the following would you expect to have the highest moment of inertia? (Assume in every case that the axis of rotation is a symmetry axis, i.e. it passes through the center of each object and is perpendicular to the flat face of the hoop or disk.)

1. A solid, uniform ball of mass 1 kg and radius 10 cm.
2. A hollow spherical shell of mass 1 kg and radius 10 cm.
3. A circular hoop or ring of mass 1 kg and radius 10 cm.
4. A uniform disk of mass 1 kg and radius 10 cm.
5. They should all be the same.

ANS: **3**—The hoop/ring has the greatest rotational inertia.

All of the objects have the same mass and radius. However, for the circular hoop, all of the mass is located a distance of 10 cm from the axis of rotation. For every other object, some of the mass is closer to the axis than 10 cm.