A ball on the end of a string is moving in a horizontal circle on a frictionless table. The other end of the string is threaded through a hole at the origin, and the string is slowly shortened by pulling it through the hole. Eventually, the radius of the circle is half of the original length. What happens to the angular speed of the ball? What happens to the linear speed? Answer both questions as quantitatively as possible. Explain your reasoning.

First question: what remains the same in this situation? Energy?

By **conservation of energy**,

(1/2)(I_1)(w_1^2) = (1/2)(I_2)(w_2^2)

Then, (1/2)(mr^2)(w_1^2) = (1/2)(m(r/2)^2)(w_1^2)(w_2) = 2w_1

w_2 = 2w_1 The angular speed will double, v_1 = rw_1. v_2 = (r/2)w_2 = rw_1

The linear velocity is thus unchanged.

Could it be angular speed?

The **angular speed is the exact same as before because angular speed is constant if there is no other torque acting on the ball. For example, if the angular speed of the ball is 5 radians per second**, it stays at 5 radians per second no matter how long or short the radius of the ball is around the axis. **However, the linear speed decreases. This is because tangential (linear) speed is reliant on the radius and the angular speed by the formula Vt = rw**, and we know w remains constant, so as the radius shrinks the tangential speed shrinks as well (I used an italicized w for the variable representing angular velocity because I can't type in Greek letters here).

How about linear speed?

The angular speed of the ball is equal to the linear speed over the radius, or length of the string. That length of the string is what is changing in this problem. **The linear speed should stay the same**, but as the string and therefore radius shorten, the constant linear speed is divided by a smaller and smaller number. This means that angular speed should consistently increase.

Could it be angular momentum?

When the radius of the circle is half of the original length, the center of mass is closer to the axis of rotation, so the moment of inertia decreases. **The angular momentum of the system is conserved, so the angular speed must increase to compensate for the decrease in the moment of inertia**. In terms of the conservation of linear momentum, the mass of the system does not change, so the linear speed does not change.

Jupiter's moon Io ejects 1 ton (=10^3 kg) of ionized sulfur and oxygen gas every second (wow!!!!). Even though Io is 400,000 km from Jupiter, this gas is swept up into Jupiter's magnetic field and rotates with the planet at Io's orbit, making a complete circuit every ten hours. The entire cloud of gas encircles Jupiter like a huge donut in space, with a total mass of about a million tons. Estimate the total angular momentum of this cloud.

Two ways to express angular momentum

Angular momentum is defined as the moment of inertia times angular velocity L = Iw. The mass and radius in the given information can be used to calculate the moment of inertia for the cloud **I = mr^2 so I =(10^3kg)(400,000km)^2 = about 1.5x10^14 kg*km^2.** Next **angular velocity can be found with the equation w = (delta theta)/(delta t)**. Since Io and the cloud complete a whole orbit around Jupiter in ten hours, (360*pi)/180 = about 6 so **w = 6rad/10hr = 0.6 rad/hr**. Finally, the moment of inertia and angular velocities can be to find the angular momentum of the gas cloud L = (1.5x10^14 kg*km^2)(0.6 rad/hr) = about (7.5x10^13 kg*km^2)/hr.

Alternatively:

Angular momentum (L) is equal to the product of the mass, radius, and velocity. The velocity is equal to the angular velocity times the radius. This means that angular momentum is equal to the mass times the radius times the angular velocity times the radius, or **L = m(r^2)(omega).** The mass of one of these clouds is 10^3 kg, the radius is 400,000 km (4 x 10^5 km), and the angular velocity is 2pi radians in 10 hours, so 1pi/5 radians per hour.

The radius squared is equal to 16 x 10^10 km^2, that times 10^3kg is 16 x 10^13 kg km^2, and that times the angular velocity is (3.2 x 10^13)pi kg km^2/hour.

Everything present, just a little hard to follow: (reminder to use ^)

Since I=mr2 , L=Iw, angular momentum can be defined by L=mr2w, where L is the angular momentum, m is the object's mass, r is the distance from the radius, and w is angular velocity. m = 103kg * 106kg = 109kg. w= 2pi radians/10 hours*1 hour/3600 s = 2pi radians/36000 s = 6/36000 radians/s = 1/6000 radians/s = 1.6 * 10-4 radians/s. r2 =4000002 = 1.6 * 1011 m2. Therefore L=(109 kg)(1.6 * 10-4radians/s)(1.6 * 1011 m2) = 2.56 * 1016 kg*m2/s.

Rearranged for my convenience!

Since I=mr^2 , L=Iw, angular momentum can be defined by L=mr^2w, where L is the angular momentum, m is the object's mass, r is the distance from the radius, and w is angular velocity.
m = 10^3kg * 10^6kg = 10^9kg.
w= 2pi radians/10 hours*1 hour/3600 s = 2pi radians/36000 s = 6/36000 radians/s =1/6000 radians/s = 1.6 * 10^-4 radians/s.
r^2 =400,000^2 = 1.6 * 10^11 m^2.

Therefore
L=(10^9 kg)(1.6 * 10-4radians/s)(1.6 * 10^11 m^2) = 2.56 * 10^16 kg*m^2/s.

Another warning about the value of units!

10^6 * 10^3 = 10^9 kg = mass of the cloud of gas. 400,000 km = 4*10^8 m
I = mr^2 = 10^9 kg * (4*10^8 m) = 4 * 10^17 kg/m

Jupiter has a radius of about 40,000 miles which would be about 70,000 km. The gas rotates with Io's orbit so it has a radius from the center of Jupiter of about 470,000 km. The circumference of a circle is pi*2r. So the distance of the gas ring is 3.14*2*470,000 = about 2,800,000 km. It travels this in 10 hours. **This means it travels 280,000 km in one hour. This would be 2.8*10^8 m/hr. Multiplied by 3,600s/hr, you get about 1*10^12 m/s.**

L = Iw = 4*10^17 kg/m * (1*10^12 m/s)/(2.8*10^8 m) = about 1.2*10^21 kg*m/s

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A gymnast vaulting through the air (okay, through a vacuum!) can change
a. angular momentum
b. rotational kinetic energy
c. translational momentum
d. all of the above
e. none of the above