

A child runs toward a stationary merry-go-round, leaps onto the platform, and catches hold of one of the bars, which makes it turn. The merry-go-round clearly has angular momentum in the final state but didn't in the initial state, but a change in angular momentum means it had to come from somewhere else. Explain how this comes about.

Key is the change in angle as measured from the center

Initial angular momentum is equal to final angular momentum. This means that there has to be some nonzero initial angular momentum. The axis of rotation has to be through the center of mass of the merry-go-round. Using this axis of rotation, the child has an initial angular momentum because the child has a tangential velocity that is not in the exact direction of the center of mass of the merry-go-round using  $\omega = v_t/r$ .

More expansive discussion

When the child is running toward the merry-go-round, they are presumably not running on a path that would take them directly through its center (the axis of rotation). This means that the child's velocity will have some non-zero tangential component. In other words, at any given instant, the child's linear velocity can be expressed as a combination of both radial and tangential components (though for the purposes of the merry-go-round's rotation, only the tangential component matters). Taking this tangential component and dividing it by the distance of the child from the axis of rotation (the child's effective radius) gives the child an angular velocity. As the child has an angular velocity, they must also have some angular momentum. When the child lands on the merry-go-round, the system as a whole must retain the same total angular momentum, so the merry-go-round (with child) turns. Essentially, relative to the merry-go-round, the linear motion can be seen as rotational motion (albeit with constantly changing radii and angular velocities).

Important: angular momentum cannot be converted into linear momentum!

The angular momentum comes from the linear momentum of the child after jumping on the merry go round.

A more extended response to work with

As we learned, linear motion is very closely related to angular motion. Objects with linear momentum intrinsically have angular momentum; this is because just like angular motion can be attributed to multiple linear velocities with respect to an axis of rotation, angular momentum also follows the same logic, except that mass also factors into the system. With respect to a defined axis, all linear motion can be described as angular motion that also experiences changes in radial direction. Therefore, a child running onto a merry go round has linear motion. From the standpoint of the axis of rotation for the merry-go-round before the child jumps on, the child has angular momentum (because he has a velocity and mass) but is also changing his radial direction relative to that axis. When he jumps on the merry-go-round, his radial direction no longer changes and all of his linear momentum is therefore translated to angular momentum. It's not that angular momentum was created in that instance, it is that linear motion and angular momentum are so intrinsically linked that simply defining an axis of rotation and removing radial change of a body in motion around that axis causes that linear motion to be described as angular motion.

If you take the vector product of 2 vectors, what angle between them produces the largest vector product? In what direction is the vector product?

Key is how perpendicular the two vectors are

Since the vector product depends on how perpendicular the two vectors are; a 90 degree angle between the two vectors would produce the largest vector product. The vector product is perpendicular to both vectors and the plane they define. It can be found using the right hand rule.

The magnitude depends on the sine of the angle

Since this a  $\text{vector product} = |A| \times |B| \times \sin(\theta)$  would be used in the situation. So the cross product is perpendicular to the two vectors: A and B. The largest angle the product produces is 90 degrees. The direction is determined by using the right hand rule and also where the point of origin is.

But how does the right-hand rule work?

A 90-degree angle between them produces the largest vector product. If you use your right hand and point towards the first vector and curl your fingers towards the second vector, the vector product will point in the direction of your thumb.

A specific example

The vector product of 2 vectors is proportional to the sin of the angle between the two vectors. This value will have the greatest value when the vectors are perpendicular, because the angle will be 90 degrees, and the sin of 90 degrees is 1, which will be greater than the sin of any other angle between the vectors. Using the right hand rule, the direction of the vectors and vector product can be found. If the two initial vectors intersect in the same flat plane, the vector product will be perpendicular to the plane. Therefore, if the first vector were pointing straight up, and the second vector was pointing 90 degrees to the right, the vector product would be pointing into the page using the right hand rule.

What do you get if you cross a mountain climber with a mountain goat?

- a. Sir Edmund Hillary Gruff
- b. Something that climbs Mt. Everest but eats its oxygen cylinder before getting back down
- c. Capricorn on the Matterhorn
- d. You can't cross them, they're scalers