The Syllogism of Studying

- The purpose of studying is to learn new things
- By definition, new things are not already know
- Hence, the first goal of studying is to identify things you don't know, not to review the things you do know

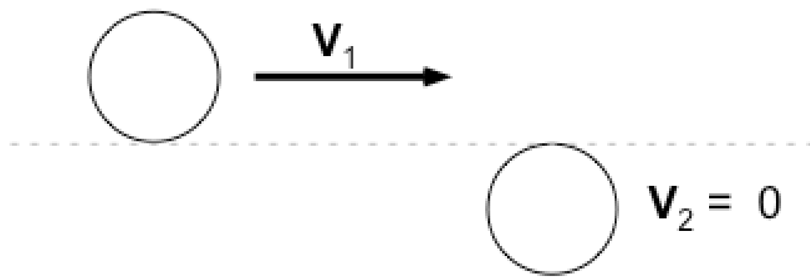
Your professor walks across the lecture hall from right to left in a straight line just in front of the board. At one point he is exactly in the middle of the screen, right in front of Izzy, who sits directly in the middle of the classroom. At that instant you (sitting where you are right now) determine the magnitude of your professor’s angular momentum to be

1. greater than what Izzy determines
2. exactly equal to what Izzy determines
3. smaller than what Izzy determines
4. zero

Your professor now walks away from the center board directly towards Izzy. You determine the direction of your professor’s angular momentum to be

1. upward
2. downward
3. left
4. right
5. zero

One hockey puck slides across a frictionless plane toward a second puck on a grazing trajectory, as shown below. Before they meet, is there any angular momentum in this situation?



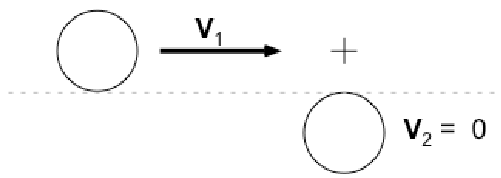
1. No, nothing is rotating
2. No, the first puck is moving but has no angular motion
3. Yes, the first puck moves about the second puck
4. The answer depends (on what?)

ANS: **4**—The answer depends on the axis of rotation.

Angular momentum is always computed around some axis. Where we place that axis determines whether or not there is angular momentum.

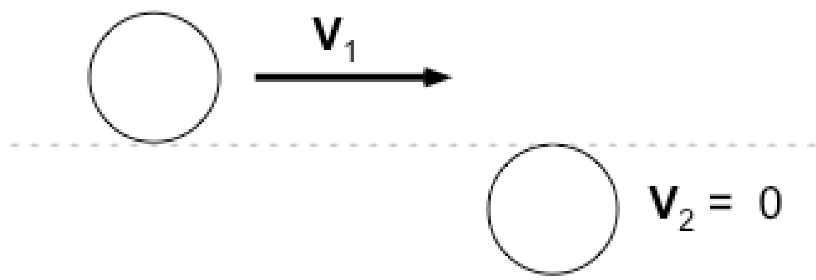
For example, if we put the axis at the center of the stationary puck, that puck has no rotation around the axis. However, the moving puck has a clockwise angular momentum around the axis, so there is a non-zero total (clockwise) angular momentum around that axis.

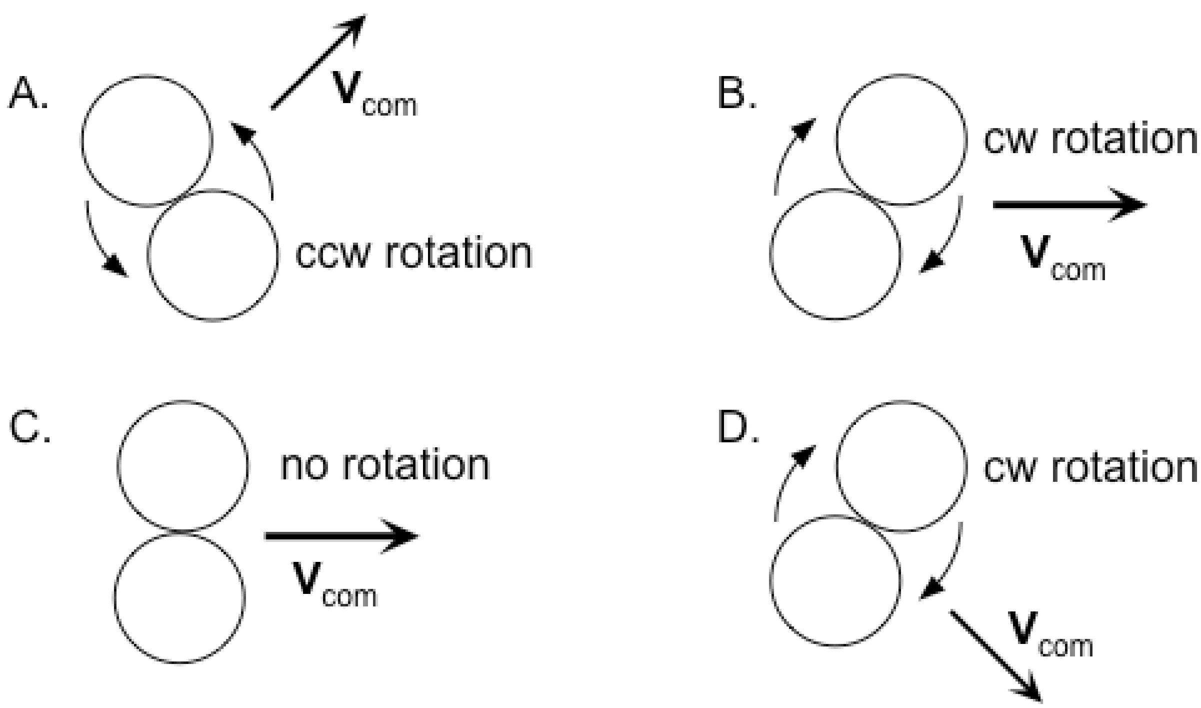
On the other hand, if we put the axis of rotation at a point on the line traced out by the center of the moving puck, but at rest with respect to the stationary puck (denoted by the “+” in the image below), then neither puck has an angular momentum around that axis and there is *no total angular momentum* around that axis.



Now let’s consider one more special axis of rotation, perpendicular to the page and passing through the system’s center of mass. This axis will move to the right as the top puck approaches the bottom one. *Relative to this axis*, the top puck will be moving to the right, while the bottom puck will be moving to the left. Both pucks have a clockwise angular momentum around this axis, so there is a total clockwise angular momentum around that axis.

Continuing with the same situation, each puck is encircled with Velcro, so they will stick together upon meeting. Which shows the outcome of the collision?





E. No linear motion, just clockwise rotation

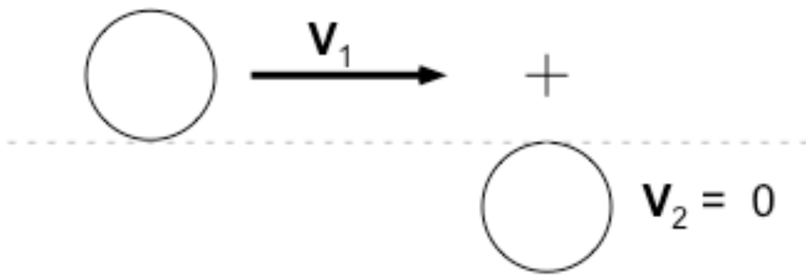
F. Other

ANS: **B**—Clockwise rotation about the center-of-mass, which moves to the right.

This is a perfectly inelastic collision. Before the collision, the system will have a total linear momentum to the right. Linear momentum is conserved in the collision, so the masses will continue moving to the right with half the initial speed of the moving puck.

Now let's consider the rotational axis to be in/out of the page, through the center of mass of the system. As we saw from the previous question, the pucks both have a clockwise angular momentum around that axis, to there is a total clockwise angular momentum around that axis before collision. There is no net external torque in the collision (all torques are internal) so the system will have the same total angular momentum after collision. Since the pucks stick together, the pair will spin around the axis through the system's center of mass.

After colliding and sticking together, the two pucks move away, spinning together about their center of mass. Which of the following correctly explains what happens? Take the point indicated by the cross to be the axis of rotation.



1. The incoming puck has no angular momentum, so there can be no rotation after they collide.
2. The incoming puck's linear momentum is converted into angular momentum.
3. After the collision, the angular momentum from spinning is equal and opposite to the angular momentum due to the motion of the center of mass.
4. Angular momentum is increased by the collision, which is possible since this is not an isolated interaction

ANS: **3**—The spinning and center-of-mass motions have equal and opposite angular momenta.

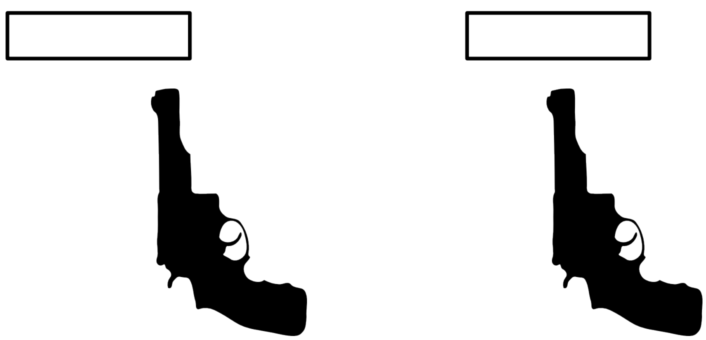
This seems really strange at first. As we established earlier, there is no initial angular momentum around the given axis, which lies along the line traced out by the center of the moving puck, and is at rest relative to the stationary puck. Angular momentum must be conserved around any axis in this problem, so after collision, the angular momentum around this axis will still be zero.

However, as we established in the previous problem, the pair of pucks will move off spinning clockwise and therefore will have a clockwise angular momentum around an axis through their center of mass. While you don't know how to calculate it, this spinning motion results in a clockwise angular momentum around the given axis.

We also established that the center of mass of the pucks will continue moving to the right. Therefore, the center of mass of the puck will have a *c*ounterclockwise angular momentum around the given axis. (This is easier to calculate if you want to try it.)

The sum of these two angular momenta, the total angular momentum around the given axis after collision, must be zero. Therefore, the two angular momenta must be equal in magnitude and opposite in direction.

Two identical blocks of wood are placed above the barrels of identical pistols. These guns are both pointed upward, but one points at the center of a block, while the other points well off the center. Therefore, the block on the left will rotate significantly once the triggers are pulled. *The bullet is embedded in the block in both cases.*



Which block will rise to a higher maximum height?

1. Right block, with bullet in center
2. Left block, with bullet at one end
3. Same height
4. Need more information

ANS: **3**—The blocks will rise to the same maximum height.

There's a lot going on in this problem, so let's be careful. We know that the bullets will have equal momentum before collision, so the blocks (with embedded bullets) will have equal momentum after collision. This gives the blocks equal *translational* kinetic energy after collision ($K_{\text{trans}} = p^2/2m$). Therefore the blocks will rise to the same height as the translational kinetic energy is converted to gravitational potential energy. *Rotational considerations have nothing to do with this result!*

There is rotation in the problem, but it does not affect the answer. Angular momentum is conserved in the collision, so the block/bullet on the left will spin and have a good deal of rotational kinetic energy. The block/bullet on the right will have no rotational kinetic energy. However, because there are no dissipative torques as the blocks rise, their rotational kinetic energies will remain constant. It is only the translational kinetic energy that is converted to gravitational potential energy.

A common misconception is to think that the blocks should have the same mechanical energy after collision because the bullets had the same mechanical energy before. This is certainly not true because these collisions are *completely inelastic*. Mechanical energy is not conserved in the collision.

The block on the left clearly has more mechanical energy after the collision than does the block on the right. Why the difference? In both cases, mechanical energy will be lost in the collision. As the bullet bores into the wood, dissipative forces between the block and bullet take energy out of the system. Apparently the spinning block, which has more mechanical energy after collision, loses less energy in the inelastic collision. Therefore, we can predict with confidence that the bullet will be embedded more deeply into the block on the right, and less deeply in the spinning block on the left. (Less displacement for a given dissipative force means less negative work done.) If we cut the blocks open we will find this to be true.

Warmup Question

If you take the vector product of 2 vectors, what angle between them produces the largest vector product? In what direction is the vector product?

ANS: The largest vector product comes when the two vectors are perpendicular, so the angle between them is 90° or $\pi/2$ radians. The direction of the vector product is perpendicular to both vectors, i.e. perpendicular to the plane spanned by the two vectors.

Warmup Question

What is the difference between a scalar product and a vector product?

ANS: Mathematically, a scalar product is a way of multiplying two vectors to get a scalar. It can be used to multiply the magnitude of one vector with the component of a second vector that is parallel to the first.

On the other hand, a vector product is a way of multiplying two vectors to get a vector. It can be used to multiply the magnitude of one vector with the component of a second vector that is perpendicular to the first.

Warmup Question

What do you get if you cross a mountain climber with a mountain goat?

1. Sir Edmund Hillarygoat Gruff
2. Something that climbs Mt. Everest but eats its oxygen cylinder before getting back down
3. Capricorn on the Matterhorn
4. You can’t cross them, they’re scalers

ANS: **4**—You can’t cross them, they’re scalers.