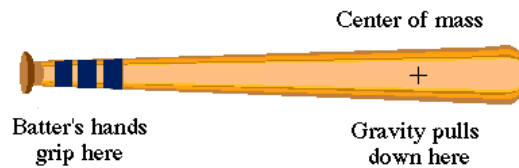


If a batter holds a baseball bat out horizontally, the center of mass is quite far from where he grips it. Discuss the forces (plural!) he must apply to keep it stationary in this position. And try to be thorough.



Balance requires considering forces and their torques

For the bat to be in static equilibrium, net force and net torque must be zero. If you consider the handle where a batter grips the bat to be the axis of rotation, the center of mass is at the opposite end of the bat. The force of gravity on the bat can cause a torque as the batter holds it horizontally because there will be a moment arm, d , from the axis of rotation. There will be no torque from the grip point, because there is no moment arm, d . Therefore, in order to keep the bat stationary, the batter must exert a force equal to the torque from the force of gravity on the bat to ensure the net force and torque exerted on the bat is zero.

Batter can't avoid exerting two forces

To keep the bat from falling, the batter must apply an upward force on the handle to prevent it from falling. This upward force from the hands would also apply a clockwise torque about the handle. Gravity would also create a clockwise torque about the handle because the center of mass is away from the axis of rotation. This means that the batter must apply a downward force behind the axis of rotation near the palms of the hands.

Good examination of how the force and lever arms interact

The forces are going to be a combination of the normal force of your hands as well as a downward force. The first force, the normal force of your hands, is positioned near the front of your fingers more towards the center of the bat. This both acts to reduce the lever arm of gravity and increase the lever arm of the next force. The downward force is applied as far back as your hand can apply it (assuming that either only one hand is holding it or your hands are acting together as a single entity). In the image above, the force of gravity would pull the bat straight down if released—with no spin. Once the upward force of the hands is applied, however, both the push given and gravity's application exert torques acting in the clockwise direction. If no other force is applied, the bat would spin until it hit the ground; however, the upward hand force is necessary to create a spinning body. The second force applied is applied downward and exerts a torque acting in the counterclockwise direction of this rotating body. The 2nd force applied by the hands needs to be much larger than the sum of the other forces because the lever arm is much smaller from the back of the hand to the front of the hand than the center of mass to the front of the hand. Once the torques applied in the clockwise and counterclockwise directions are equal, the bat is stationary.

Estimate the magnitude of the torque exerted by the batter in the discussion question. Explain your reasoning.

It all depends on the axis of rotation chosen

I would estimate that the distance from the handle to the center of mass on the bat would be around 0.5 m. The weight of a baseball bat is probably around 2 kg. Applying the equation $T = F \cdot r$, the torque applied by gravity would be about 10 N·m ($20\text{N} \cdot 0.5\text{m}$). Therefore, the torque applied by the person holding the handle would have to be 10 N·m.

...so if you make a clever choice, the answer is trivial

The axis of rotation goes through the center of mass of the bat.

Since the axis of rotation goes through the center of mass of the bat, the moment arm of the gravity would have a length of zero. Therefore, the torque of gravity would also be 0. Gravity would still act as a force, and the batter would have to apply an equal and opposite force to keep the bat from falling, but the torque of gravity would be zero.

Since the torque of gravity is zero, the batter cannot exert a torque on the bat, or the static equilibrium of the situation would not stand true.

But you can also calculate the individual forces of the two hands!

The total torque exerted by the batter must be equal to 0 if we are to allow the axis of rotation to be through the center of mass. This is because gravity would then be exerting 0 N·m of torque. Thus, the two hands must also exert a total of 0 N·m of torque. However, each individual hand's torque can also be calculated (again, with the axis of rotation through the center of mass). We'll allow the leftmost hand (according to the diagram) to be F_{N1} and the rightmost hand (according to the diagram) be F_{N2} . We'll let up be the positive direction. Now, $F_{N1} + F_{N2} + F_g = 0$ N, so $F_{N1} + F_{N2} = -F_g$. If the mass of the bat is 1 kg, this means $F_{N1} + F_{N2} = 9.8$ N, or $F_{N1} = 9.8$ N - F_{N2} . Now, the torques must also add to zero. So, $F_{N1}(d_1) + F_{N2}(d_2) = 0$. Substitution yields $(9.8 - F_{N2})(d_1) + F_{N2}(d_2) = 0$, or $F_{N2} = -(9.8d_1)/(d_2 - d_1)$. If a baseball bat is 1 m, and the leftmost hand is further from the center of mass, then let $d_1 = 0.7$ m and $d_2 = 0.65$ m. Then F_{N2} approximately equals -7 N·m / $(-0.05$ m), or $F_{N2} = 140$ N (140 N up). Then F_{N1} must equal approximately -130 N (downward), as $F_{N1} + F_{N2} = 9.8$ N. Then $\tau_1 = 130$ N (0.7 m), or around 100 N·m counterclockwise. Then τ_2 must be 100 N·m clockwise.

Where should you choose the axis for a body in static equilibrium?

- a. The center of mass
- b. The geometric center
- c. The axis of rotation if the body started rotating
- d. The far left end
- e. The far right end
- f. Wherever makes the problem simple