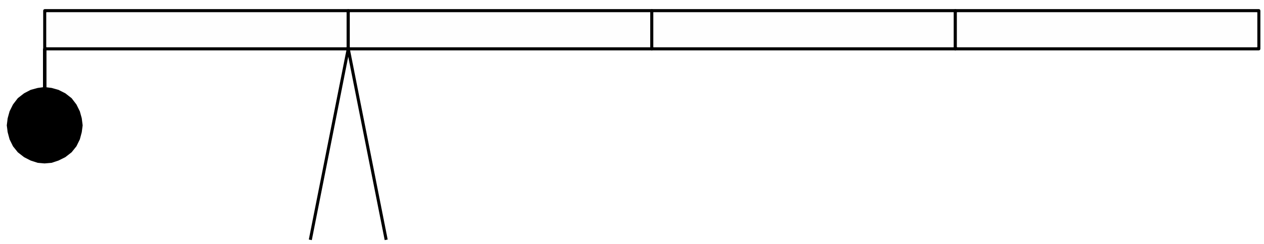
Static Equilibrium

The most important information exchanged in a class is that passed from the students to the professor, not the reverse. In the absence of that, the students might as well not be there.

A 1 N rock is suspended by a string from one end of a uniform 1 m measuring stick. If the stick is balanced on a support at the 0.25 m mark, what is the weight of the stick?



1. 0 N
2. 0.25 N
3. 0.5 N
4. 1 N
5. 2 N
6. 4 N
7. impossible to determine

ANS: **4**—The stick weighs 1 N.

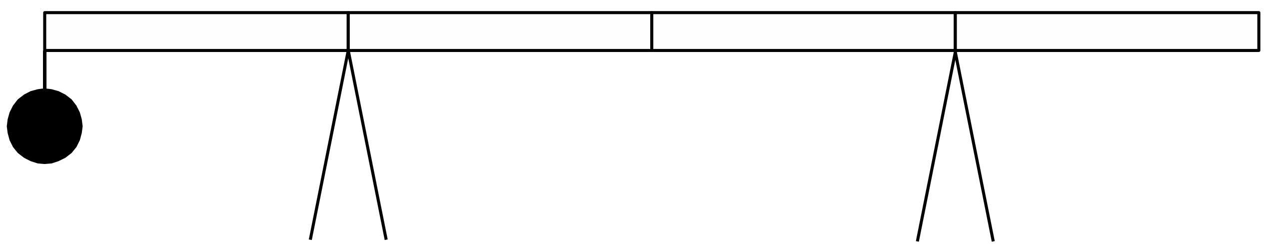
For the purposes of static equilibrium, we can treat the meter stick as if all of the gravitational force on it were applied at the center-of-gravity. In fact, this is the definition of center-of-gravity. Break up the stick into pieces and compute the torque about any axis you choose due to the gravitational force on each piece. Add up all of these torques. If you instead chose to pretend that all of the mass of the stick were concentrated at a single point that will produce the same net torque, then the location of this point is the center of gravity.

For a symmetrical object like a uniform meter stick, the center of gravity will be located at the geometric center of the stick. In this problem, the center of gravity is located 0.5 m from the left end of the stick and therefore 0.25 m to the right of the support. If we choose the support to be our axis of (non)-rotation, we see that both the force due to the hanging mass and the force due to the weight of the stick have equal-magnitude lever arms (0.25 m) in both cases.

Static equilibrium requires that the net torque around any axis must be zero. Choosing the support as our axis, we see that the hanging mass exerts a $0.25\,\mathrm{N \cdot m}$ counter-clockwise torque around it. That means that the torque due to the weight of the stick must be $0.25\,\mathrm{N \cdot m}$ clockwise. Since the lever arm for this force is 0.25 m, we know that the weight of the stick must be 1 N. (The specific lengths of the lever arms do not matter. Equal lever arms require equal forces to produce the same torque.)

Although the problem did not not ask for it, we can use the condition for static equilibrium to compute the force applied by the support. The rock and the meter stick together exert a total of 2 N downward on the stick. Therefore, the support must exert an upward force of 2 N on the stick.

The same 1 N rock and 1 N meter stick as before are now supported by an *additional* support, as shown. How much force is exerted on the stick by the right support?

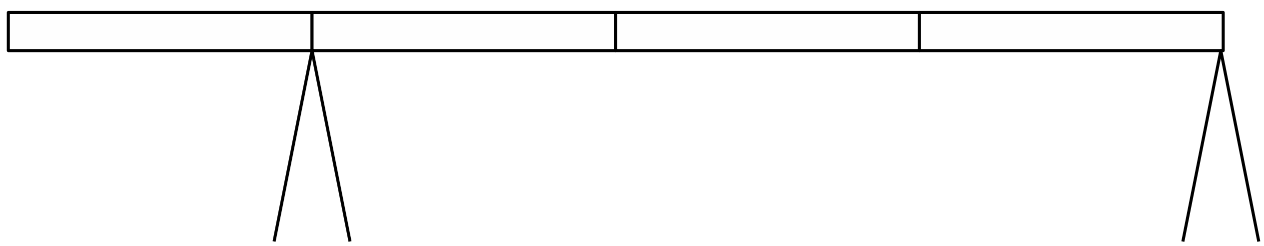


1. 0 N
2. 0.25 N
3. 0.5 N
4. 1 N
5. 2 N
6. 4 N
7. impossible to determine

ANS: **1**—The right-hand support applies no force on the stick.

The two downward forces have not changed in this problem. Let’s again choose to put the axis of (non)-rotation to be the point of the left-hand support. The two downward forces create opposite torques around this axis. If the right-hand support exerted a force on the meter stick, it would also create an unbalanced torque, giving a non-zero net torque on the meter stick. For the problem to be static, this is not possible.

A 1 N meter stick rests on two supports, one at the 0.25 m mark and the other at the 1 m mark. What force does the right-hand support exert on the stick?

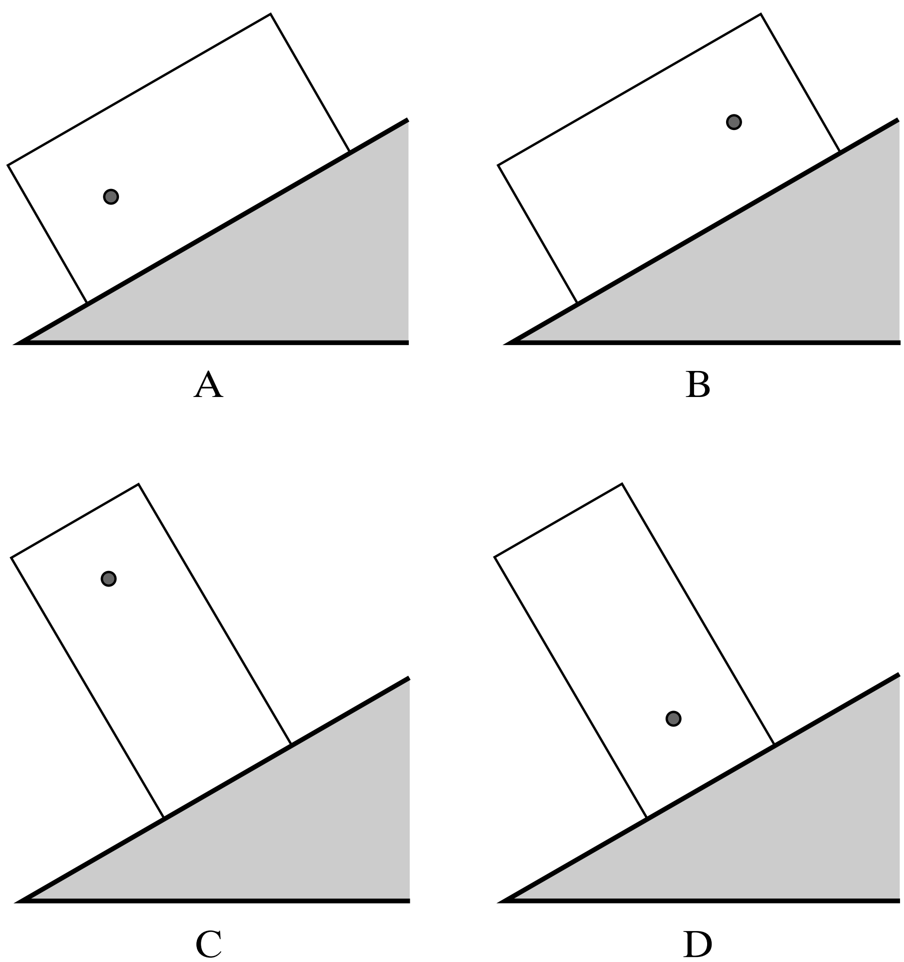


1. 0 N
2. 1/3 N
3. 1/2 N
4. 2/3 N
5. 1 N
6. impossible to determine

ANS: **2**—The right-hand support applies a force of $1/3\,\mathrm{N}$ to the stick.

Let's leave the axis of (non)rotation at the left-hand support. The weight of the meter stick creates a $(1\,\mathrm{N})(0.25\,\mathrm{m}) = 0.25\,\mathrm{N\,m}$ clockwise torque around this axis. The right-hand support, therefore, must also create a $0.25\,\mathrm{N\,m}$ counter-clockwise torque around the axis. This force has a moment arm of 0.75m, so the applied force must be $(0.25\mathrm{N\,m})/(0.75\mathrm{m}) = 1/3\,\mathrm{N}$ upward. More simply, the moment arm for the right-hand support is three times longer than the moment arm for the stick's weight, so the force applied by the support must be three times smaller than the weight of the stick.

A box, with its center-of-mass off-center as indicated by the dot, is placed on an inclined plane. In which of the four orientations shown, if any, does the box tip over?



ANS: Box **3** will tip over.

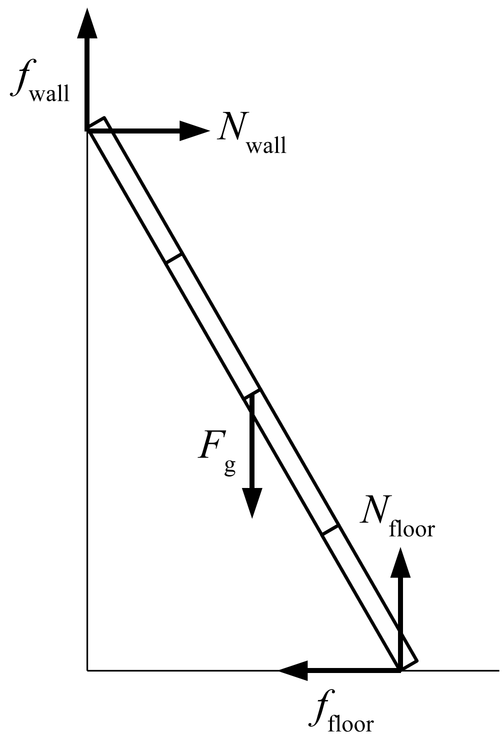
There are three forces on each block: the weight of the block, directed downward at the center-of-mass (indicated by the point); the normal force, which points perpendicular to the incline (up and to the left); and (possibly) friction, which points along the surface of the incline. The most obvious axis to consider is the bottom corner of the box. This choice guarantees that there will be no torque around this axis due to friction. It also ensures that the torque due to the normal force, which will be applied somewhere to the right of the axis, will always be counter-clockwise. In cases #1, #2, and #4, the weight of the box produces a clockwise torque which can, in principle, balance the torque due to the normal force. This allows for static equilibrium. In case #3, the weight produces a counter-clockwise torque which adds to the counterclockwise torque due to the normal force.

The only way to keep box #3 from not tipping over is to attach it (with a nail or some adhesive) to the plane. This allows there to be an attractive force between the box and plane, which would produce a clockwise torque to counter-balance the torque due to the weight.

Note: It can be shown that, if the box does not tip over, we can model the friction and normal forces exerted on the box by the incline as being applied at the point of contact directly below the center of gravity of the box. In that case, it is easy to see by answer #3 will tip over. There is no point of contact between box and incline that is directly below the center of gravity, so the necessary force cannot be applied by the surface on the box.

You should try to prove this to yourself. If you can’t or if you want to check your work, come see me.

A ladder leans motionless against the wall with the forces shown (arrows do not indicate force magnitudes). Could the ladder remain motionless if one or both of the frictional forces were removed? (Imagine that each frictional coefficient is as large as you need.)



1. No, both frictional forces must be present.
2. Friction at the wall can be zero.
3. Friction at the floor can be zero.
4. Either frictional force can be removed, but not both.
5. Both frictional forces can be removed.

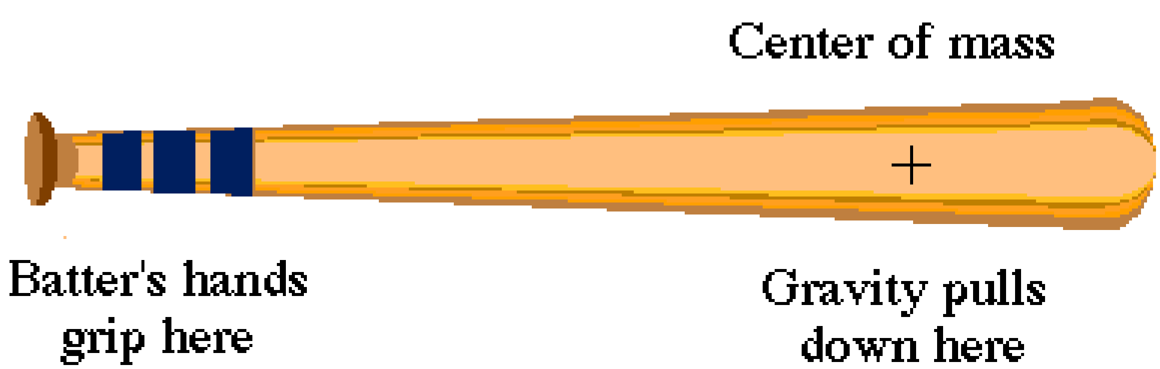
ANS: **2**—Friction at the wall can be zero.

If you remove the upward friction at the wall, you still have upward and downward forces that balance, and leftward and rightward forces that balance, allowing you to have zero net force. If you put the axis of (non)rotation at the floor contact point, you also have the clockwise torque due to N_{wall} balancing the counter-clockwise torque due to F_g, allowing you to have zero net torque. Therefore, the ladder can balance.

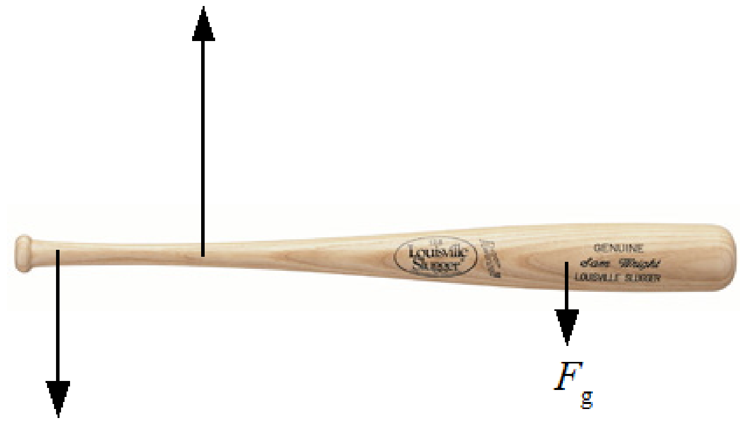
If, on the other hand, you remove the leftward friction force at the floor, you cannot have zero net force. Friction at the floor must be non-zero to balance the rightward normal force at the wall. Why must there be a rightward normal force at the wall? Again, put the pivot at the floor contact point. Gravity creates a counter-clockwise torque, while the normal force and friction at the wall create clockwise torques that can counter-balance it. Conceivably, the friction force at the wall could be large enough to balance the gravitational torque even if N_{wall} were zero. However, the only way you can have friction at the wall is if there is a non-zero normal force of contact.

Warmup Question

If a batter holds a baseball bat out horizontally, the center of mass is quite far from where he grips it. Discuss the forces he must apply to keep it stationary in this position. And try to be thorough. (Note: you can experiment with this yourself by trying to hold a pencil level with your fingers at one end, far from the center. But answer in terms of the bat.)



ANS: You must apply two forces, one upward and one downward (with the upward force applied between the downward force and the bat's center of gravity). The upward force is necessary to counter-balance the downward force of gravity on the bat.



Static equilibrium also requires zero net torque. We are free to choose the axis of (non)-rotation anywhere we would like. Let's put it at the point in the handgrip where the upward force is applied (the middle force in the picture). This force produces no torque (the lever arm is zero). The weight of the bat, however, creates a definite clockwise torque. Therefore, if these were the only two forces, there would be a net clockwise torque on the bat. We must have a counter-clockwise torque that counter-balances that.

We can achieve this by exerting a downward force to the left of the axis. This will counter-balance the gravitational torque but, because the lever arm is smaller, this force must be larger than the weight of the bat. This will give zero net torque, but in order to still have zero net force, the upward force on the handgrip must be the sum of this force and the bat's weight, so it is larger than the other two.

Warmup Question

Estimate the magnitude of the torque exerted by the batter in the discussion question. Explain your reasoning.

ANS: The answer will depend a great deal on how you grip it. I'll use the picture I drew above as a guide. Let's assume that the bat has a mass of 1 kg, giving it a weight of around 10 kg (around 2 lb). Put the axis at the location of the upward force. By eye, I estimate that the distance from the center of mass to the axis is twice as far as the distance of the (downward) left-hand force to the axis. This means that the left-hand force should be around 20 N (4 lb). To balance the forces, the upward force should be 30 N (around 6 lb).

Warmup Question

Where should you chose the axis for a body in static equilibrium?

1. the center of mass
2. the geometric center
3. the axis of rotation if the body started rotating
4. the far left end
5. the far right end
6. wherever makes the problem simple

ANS: **6**—You should put the axis wherever it makes the problem simple.

In reality, you can put the axis of “non-rotation” anywhere you want, but it is a good idea to put it where it makes the problem simple. For example, if your problem has unknown forces, if you put the axis of non-rotation at the location of one of the unknown forces, it reduces the number of forces for which you need to compute torques. If you have two unknown forces and you are only interested in finding one of them, it’s a good idea to put the axis at the location of the other force. Of course you don’t *have* to put the axis where it makes the problem simple. You’re free to make problems harder than they need to be if you would like.