Simple Harmonic Motion

Being given the answer to a challenging problem is like being told a great twist ending without watching the movie or reading the book. You can't ever go back and recover that sense of discovery.

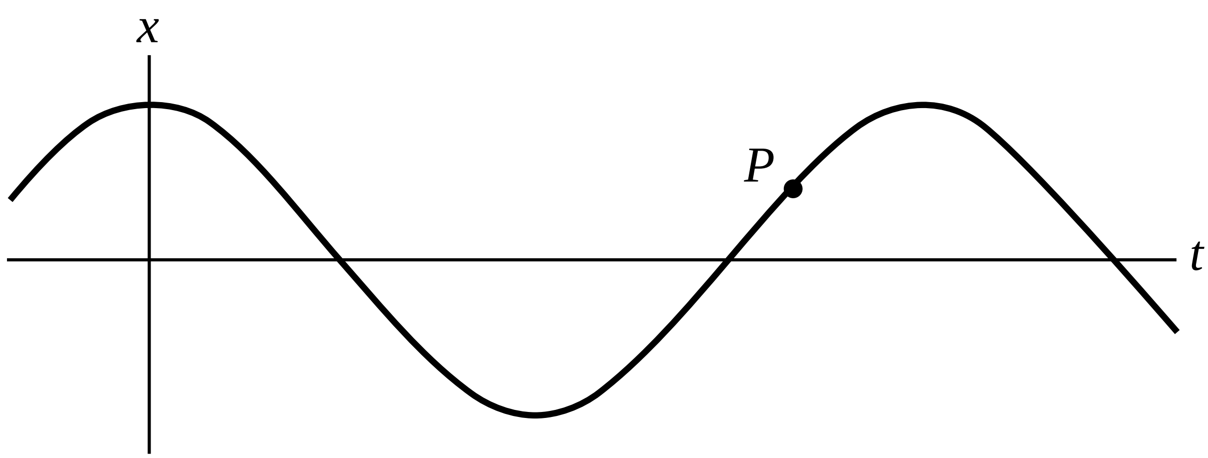
An object can oscillate around

1. any equilibrium point.
2. any stable equilibrium point.
3. certain stable equilibrium points.
4. any point, provided the forces exerted on it obey Hooke’s law.
5. any point.

ANS: **2**—an object can oscillate around any stable equilibrium point.

By definition, a stable equilibrium point is one where, if you displace the system away from the point, the resulting forces will try to return the system to that point. An unstable equilibrium point is one where if the system is displaced from it, the resulting forces will try to keep moving it away from that point.

A mass attached to a spring oscillates back and forth as indicated in the position vs. time plot below. At point *P*, the mass has



1. positive velocity and positive acceleration.
2. positive velocity and negative acceleration.
3. positive velocity and zero acceleration.
4. negative velocity and positive acceleration.
5. negative velocity and negative acceleration.
6. negative velocity and zero acceleration.
7. zero velocity but is accelerating (positively or negatively).
8. zero velocity and zero acceleration.

ANS: **2**—the mass has positive velocity and negative acceleration.

The velocity is the slope of the line tangent to the point on the position vs. time graph. At point P on the graph the velocity is positive because the mass is moving away from the equilibrium position.

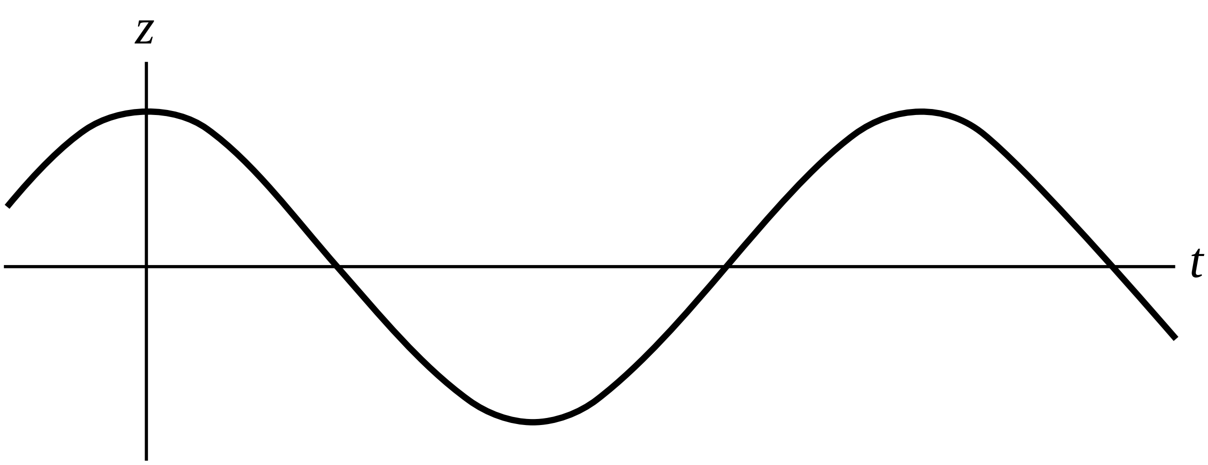
Acceleration is the rate of change of velocity. On the position graph this is represented by curvature: “concave up” indicates a positive acceleration, while “concave down” indicates a negative acceleration. At point P, the graph curves downward. You can also determine the acceleration by examining the rate of change of velocity. Before point P, the velocity was positive. After point P, the velocity was less positive, so the change in velocity was negative.

Finally, you can also think of this by looking directly at the equation for simple harmonic motion. The acceleration of the mass is $a = -\omega^2 x$, where x is the displacement from equilibrium, and ω is the angular frequency of the oscillation. Therefore, the acceleration always points opposite the displacement. At point P, x is positive so a is negative.

A mass suspended from a spring is oscillating up and down, tracing the position vs. time graph below. Consider two possibilities:

(i) at some point during the oscillation the mass has zero velocity but is accelerating (positively or negatively);

(ii) at some point during the oscillation the mass has zero velocity and zero acceleration.



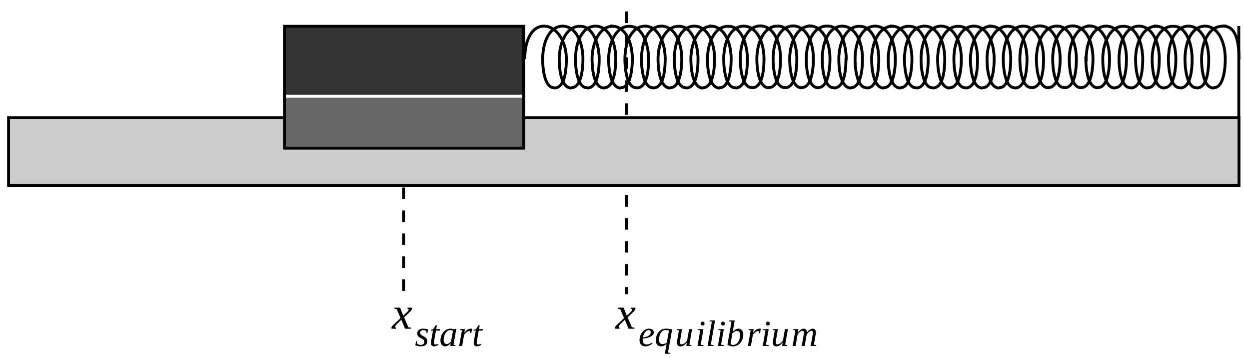
1. Both occur sometime during the oscillation.
2. Neither occurs during the oscillation.
3. Only (i) occurs.
4. Only (ii) occurs.

ANS: **3**—only (i) occurs.

The velocity of the mass is zero at the turning points, where the displacement from equilibrium is greatest (most positive or most negative). At these points, the acceleration is definitely not zero. On the contrary, the acceleration is greatest (most positive or most negative) when velocity is zero.

Conversely, the points where acceleration is zero are when the displacement is zero, i.e. when the graph crosses the horizontal axis. At these points, the speed is greatest.

A glider on a frictionless air track is attached to a wall with a spring as shown. The glider is pulled 10 cm to the left of its equilibrium position and released from rest. It takes 0.5 s to get back to the equilibrium point. If instead the mass is pulled 20 cm to the left (twice as far as before) and then released from rest, the amount of time it takes to get back to the equilibrium point is

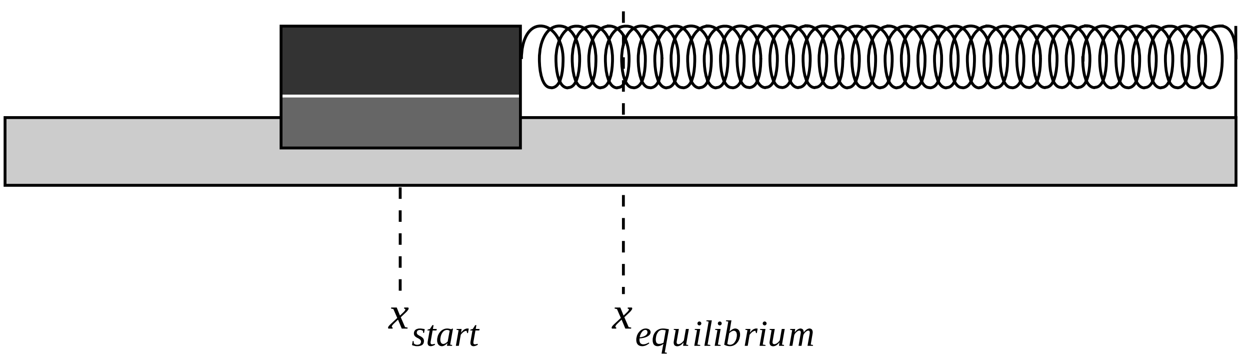


1. exactly 1.0 s
2. between 0.5 s and 1.0 s
3. exactly 0.5 s
4. between 0.25 s and 0.5 s
5. exactly 0.25 s
6. Need more information

ANS: **3**—It takes exactly 0.5 s to get back to the equilibrium point.

The time it takes to get from maximum displacement to the equilibrium position is 1/4 of the oscillation period. The period of oscillation of a harmonic oscillator is independent of amplitude. If you start it with a greater amplitude, the mass will have to travel farther, but the forces that pull it will be larger in precisely the right amount to ensure that the period (time for one oscillation) will not change.

A glider/spring system on an air track is found to have a total energy of 16 J when the amplitude is 10 cm. While oscillating with that amplitude, what is the spring potential energy when the displacement from equilibrium is 5.0 cm (half the amplitude)?

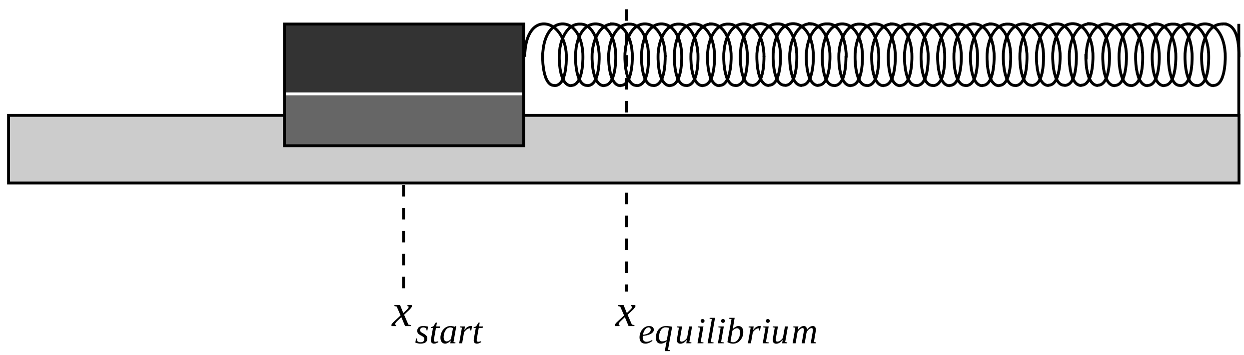


1. 1 J
2. 4 J
3. 8 J
4. 12 J
5. 16 J
6. 32 J
7. Need more information

ANS: **2**—The spring potential energy will be 4 J.

The total energy of the oscillator is equal to the elastic potential energy when the spring is at its amplitude (maximum displacement): $E = \frac{1}{2}kA^2$. When the mass is at a position of half its amplitude, $x = A/2$, its elastic potential energy is 1/4 of its total energy: $U_E = \frac{1}{2}kx^2 = \frac{1}{2}k(A/2)^2 = E/4$.

A glider/spring system on an air track is found to have a total energy of 16 J when the amplitude is 10 cm. It is now pulled back farther at the start, so the amplitude is 20 cm (twice as large as before). While oscillating with that amplitude, what is the spring potential energy when the displacement from equilibrium is 5.0 cm?

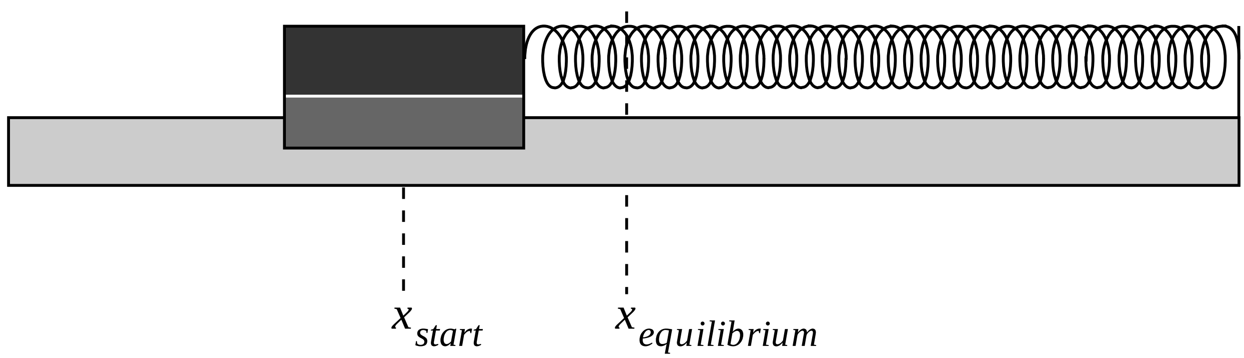


1. 1 J
2. 4 J
3. 8 J
4. 12 J
5. 16 J
6. 32 J
7. Need more information

ANS: **2**—The spring potential energy will be 4 J.

The total energy is now $(16\,\mathrm{J}) \times 4 = 64\,\mathrm{J}$, but the displacement $x = 5\,\mathrm{cm}$ is 1/4 of the amplitude, which makes the elastic potential energy at this point 1/16 of the total energy, or 4 J. Alternatively, you can realize that even though the total energy is larger, the elastic potential energy at $x = 10\,\mathrm{cm}$ is still 16 J, so the elastic potential energy at $x = 5\,\mathrm{cm}$ should still be 1/4 of that.

A glider/spring system on an air track is found to have a total energy of 16 J when the amplitude is 10 cm. The maximum speed is found to be 1.0 m/s. What is the speed when the displacement from equilibrium is 5.0 cm, i.e., half the amplitude?



1. $1.0\,\text{m/s}$
2. $\sqrt{0.75}\,\text{m/s}$
3. $0.75\,\text{m/s}$
4. $\sqrt{0.5}\,\text{m/s}$
5. $0.5\,\text{m/s}$
6. $\sqrt{0.25}\,\text{m/s}$
7. $0.25\,\text{m/s}$
8. Need more information

ANS: **2**—The speed is $v = \sqrt{0.75}\,\mathrm{m/s}$.

At 5 cm, the displacement from equilibrium is half the amplitude, so the potential energy at this point is 1/4 of the maximum potential energy, or 1/4 of the total energy. This means that the kinetic energy is 3/4 of the maximum kinetic energy, which corresponds to a speed of 1.0 m/s. This means that the speed of the mass at half the amplitude is $\sqrt{3/4}\,(1.0\,\mathrm{m/s}) = \sqrt{0.75}\,\mathrm{m/s}$.

Warmup Question

A simple harmonic oscillator is characterized by three independent parameters: amplitude, angular frequency, and that phase constant thing. Now imagine a spring that is oscillating back and forth through a certain range of motion. If you start the oscillations over again but pull back twice as far before releasing, how does the time for each oscillation change from its initial value?

ANS: The time for oscillation, or the period, will not change. Remember that the period of an oscillation depends on the angular frequency, ω: $T = 2\pi/\omega$. The angular frequency, in turn, depends only on the mass and spring constant of the oscillator.

How can this be? Doesn’t an oscillator with greater amplitude have to move through a greater distance for each oscillation? Yes, but remember that the force applied by the spring is greater when the displacement is greater. With greater amplitude, the acceleration is greater and speeds up the motion just such that the oscillator will cover the larger distance in exactly the same amount of time!

Warmup Question

A 1.0 N weight is hung vertically from a spring, which stretches 10 cm. The spring is now reoriented horizontally with the same mass on a flat, frictionless surface, and the mass is set into oscillation. Estimate how much time is required for the mass to complete one full oscillation. Explain your chain of logic, and yes, I'm expecting a numerical answer (though an approximate one!).

ANS: The $1.0\,\mathrm{N}$ weight hangs, so the upward pulling force of the spring when it is stretched $10\,\mathrm{cm} = 0.10\,\mathrm{m}$ is $1.0\,\mathrm{N}$. From Hooke's law, the spring constant is $k = F/x = 1.0\,\mathrm{N}/0.1\,\mathrm{m} = 10\,\mathrm{N/m}$.
Now we lay the spring/mass system on its side on a frictionless table and allow it to undergo simple harmonic motion. The mass of the object is $1.0\,\mathrm{N}/9.8\,\mathrm{N/kg} \approx 0.1\,\mathrm{kg}$. The angular frequency of the oscillator is $\omega = \sqrt{k/m} = \sqrt{10\,\mathrm{N/m}/0.1\,\mathrm{kg}} \approx 10\,\mathrm{rad/s}$. The period of oscillation is $T = 2\pi/\omega \approx 2\pi/\left(3\,\mathrm{rad/s}\right) \approx 2\,\mathrm{s}$.

Warmup Question

Which of the following parameters is independent of all the others?

1. period
2. frequency
3. angular frequency
4. amplitude
5. The above are all independent of each other.

ANS: **4**—The amplitude is independent of the other quantities.

Period and both kinds of frequency are easily related (directly or inversely proportional) to each other. The period or frequency of a harmonic oscillator, on the other hand, is independent of amplitude.