Pendulum Motion

Why science teachers should not be given playground duty.

Consider a simple pendulum consisting of a small, dense plumb bob of mass m suspended from a string. Which of the following statements are true?

1. The farther the bob is from its equilibrium position, the larger the restoring force.
2. The farther the bob is from its equilibrium position, the longer it takes to swing back.
3. both of the above are true
4. neither of the above are true

ANS: **1**—The farther from equilibrium, the larger the restoring force.

The restoring force is the component of the gravitational force on the bob that is tangent to the circular path of the bob. As you pull the bob farther from equilibrium, the tangent to the curve becomes more vertical and the restoring force increases. The distance the pendulum swings through in one period also increases. At least for small angles (less than 20° or so) the force and swing distance increase in such a way that the period does not change. (So answer #2 is not correct except for large amplitudes.)

Consider a simple pendulum consisting of a small, dense plumb bob of mass m suspended from a string. Which of the following statements are true?

1. The larger the mass of the bob, the larger the restoring force.
2. The larger the mass of the bob, the longer it takes to swing back.
3. both of the above are true
4. neither of the above are true

ANS: **1**—The larger the mass of the bob, the larger the restoring force.

The restoring force is a component of he gravitational force on the bob, so it is proportional to the bob's mass. Because the restoring force is proportional to the mass, the acceleration of the bob does not change as we increase the mass. This means that the oscillation period is independent of the mass of the bob. (This is analogous to the fact that objects fall with the same acceleration, regardless of mass.)

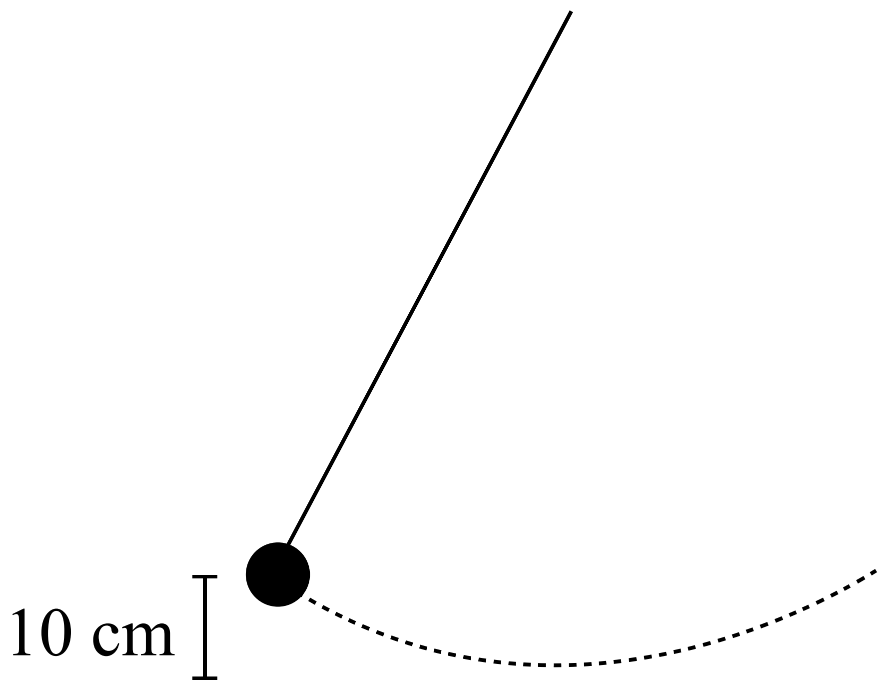
Consider a simple pendulum consisting of a small, dense plumb bob of mass *m* suspended from a string. Which of the following statements are true?

1. The longer the string, the larger the restoring force (for a given angle).
2. The longer the string, the longer it takes to swing back.
3. both of the above are true
4. neither of the above are true

ANS: **2**—The longer the string, the longer it takes to swing back.

For a given angle, the distance the bob swings in one period is greater for greater pendulum length, but the restoring force is not any stronger. Therefore, the period increases with length.

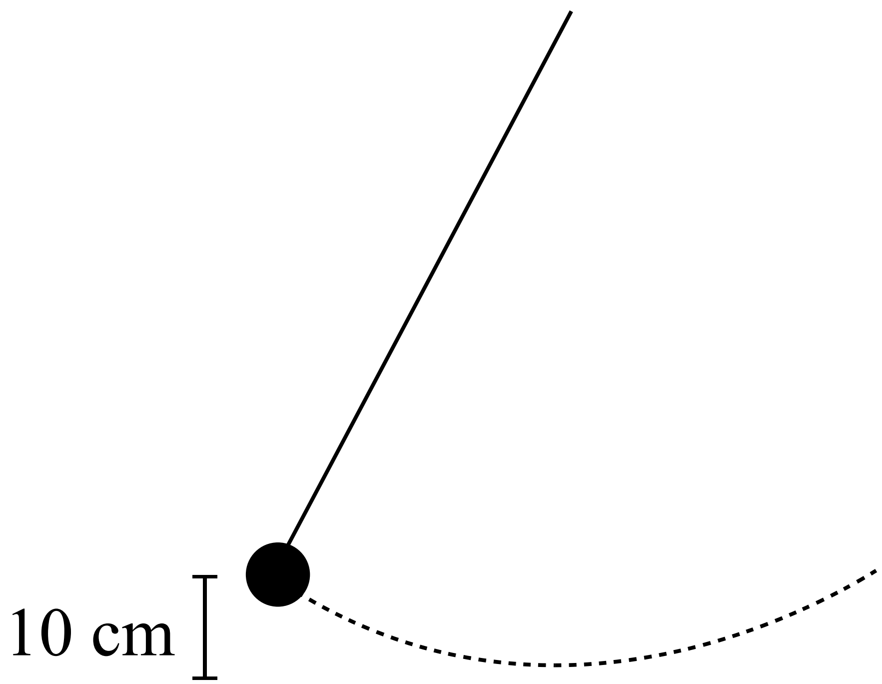
A pendulum bob of mass 1.0 kg is displaced from equilibrium, then released from rest at the point shown. What is the magnitude of the work done by the string on the bob between there and the equilibrium position?



1. Zero
2. 0.1 J
3. 1.0 J
4. 10 J
5. Can’t be determined

ANS: **1**—The force the string exerts on the bob is perpendicular to the bob’s displacement, so it does no work.

A pendulum bob of mass 1.0 kg is displaced from equilibrium, then released from rest at the point shown. What is the magnitude of the work done by gravity on the bob between there and the equilibrium position?



1. Zero
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3. 1.0 J
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ANS: **3**—$W_g = 1.0\,\mathrm{J}$

The gravitational force only does work on the bob's vertical displacement. The work is $(10\,\mathrm{N})(0.1\,\mathrm{m}) = 1.0\,\mathrm{J}$ (using $g \approx 10\,\mathrm{N/kg}$).

Alternatively, you should recognize that the work done by gravity is the negative of the change in the bob's potential energy: $-mgh = -(10\,\mathrm{N})(-0.1\,\mathrm{m})$.

For a swinging simple pendulum, at what point is the acceleration of the bob zero?

1. At its maximum angle
2. At the equilibrium point
3. Somewhere else
4. Never

ANS: **4**—The bob’s acceleration is never zero.

At the highest points of the oscillation (the turning points) the radial acceleration will be zero, but the tangential acceleration will be non-zero. At the lowest point the tangential acceleration will be zero, but the radial acceleration will be non-zero. At all other points, neither the radial nor the tangential accelerations will be zero. This leads to an interesting result: the net force on a swinging pendulum is never zero.

The amplitude of a real (not ideal) harmonic oscillator will decrease over time. This effect can be explained by

1. the action of gravity on the oscillator.
2. the action of dissipative forces on the oscillator.
3. the action of a driving force that varies with the natural frequency of the oscillator.

ANS: **2**—Dissipative forces decrease the amplitude.

The amplitude determines the total energy of a pendulum. Dissipative forces do negative work, reducing the mechanical energy of the pendulum, and therefore its amplitude.

A person sits on a swing, which oscillates back and forth at its natural frequency. If, instead, two people sit on the swing, the natural frequency of the swing is

1. greater.
2. the same.
3. smaller.

ANS: **2**—The period (and therefore frequency) of a pendulum is independent of mass.

A grandfather clock operates by the motion of a pendulum. It is designed to "tick a second" every time the pendulum swings from one side to the other. Suppose a clock is calibrated correctly, but then a mischievous child slides the bob of the pendulum downward on the oscillating rod. The grandfather clock will now run

1. fast.
2. correctly.
3. slow.

ANS: **3**—The period of a pendulum increases as you increase its length. This makes the time between successive “ticks” longer and therefore the clock will run slow.

A grandfather clock operates by the motion of a pendulum. It is designed to “tick a second” every time the pendulum swings from one side to the other. Suppose a grandfather clock is calibrated correctly at sea level, and then is taken to the top of a very tall mountain. The grandfather clock will now run

1. fast.
2. correctly.
3. slow.

ANS: **3**—The clock will run slow.

On a high mountain, the gravitational field strength is (slightly) lower than at sea level. This ensures that the restoring force is lower and the pendulum period longer at the top of the mountain.

Warmup Question

At the Smithsonian Museum of Natural Science, an impressive Foucault pendulum hangs from the third floor and all the way down to the basement. Imagine that we had a similar one hanging in the atrium of Stephens Science Center. Estimate the amount of time it would take to complete a full swing.

The period of a pendulum is given by

$$T = 2\pi * \sqrt{\frac{l}{g}}$$

where l is the length of pendulum and g is the gravitational field. For three stories at 3 meters each, *l* is 9 meters, which makes the period

$$T = 2\pi * \sqrt{\frac{9\text{ m}}{9.8\text{ m/s}^2}}$$

where for convenience I've scandalously replaced N/kg (the proper units for g) with the mathematically equivalent m/s^2. Canceling the meters and rectifying the fraction in the denominator gives

$$T = 2\pi * \sqrt{\frac{9\text{ s}^2}{9.8}}$$

The root is a shade under 1 s, while 2π is a shade over π, so within the proper spirit of estimation, the product is 6s.

Warmup Question

A 0.25 m pendulum has a period of approximately 1.0 s. Why then do you think that the pendulum in grandfather clocks are around 1 m long?

ANS: The four-times longer pendulum will have a period $\sqrt{4} = 2$ times longer than that of a $0.25\,\mathrm{m}$ pendulum, or around $[2] s$. A grandfather clock with a period of $2\,\mathrm{s}$ will tick off 1 second each time the pendulum swings from one side to the other, or half the period.

Warmup Question

What condition must be satisfied for a pendulum to execute simple harmonic motion, at least approximately?

1. The swinging mass must be concentrated in a small volume.
2. The string must not have a significant mass.
3. The maximum angle of the swing must be fairly small.
4. All of the above must be satisfied.

Answer: 3. The maximum angle must be fairly small. This remarkable fact is a consequence of the fact that every restoring force approximately obeys Hooke's law for small enough departures from equilibrium.