By shaking one end of a taut string, a single pulse is generated. The traveling pulse carries

1. mass.
2. energy.
3. momentum.
4. energy and momentum.
5. mass and energy.
6. all three.
7. none of these.

Answer: 4. A traveling wave does not involve any transport of mass (even though mass gets displaced, no mass actually travels along with the wave from one point in space to another). By shaking one end of a rope, however, one can get the other end to move, and when that end is put in motion, it has both momentum and energy. So when the wave travels down the rope, it carries both momentum and energy.

Index: propagation of waves

A wave is described by the function

$$f(x, t) = A \sin(4x + 7t)$$

where x is in meters and t is in seconds. What is its velocity?

1. + 7 m/s
2. - 7 m/s
3. + 4/7 m/s
4. - 4/7 m/s
5. + 7/4 m/s
6. - 7/4 m/s

You wish to study the behavior of traveling periodic waves on a single, uniform string tied between two posts. Although the physical properties of the string are not changed, the second wave you observe has a higher frequency than the first one. How do the velocities of two waves compare?

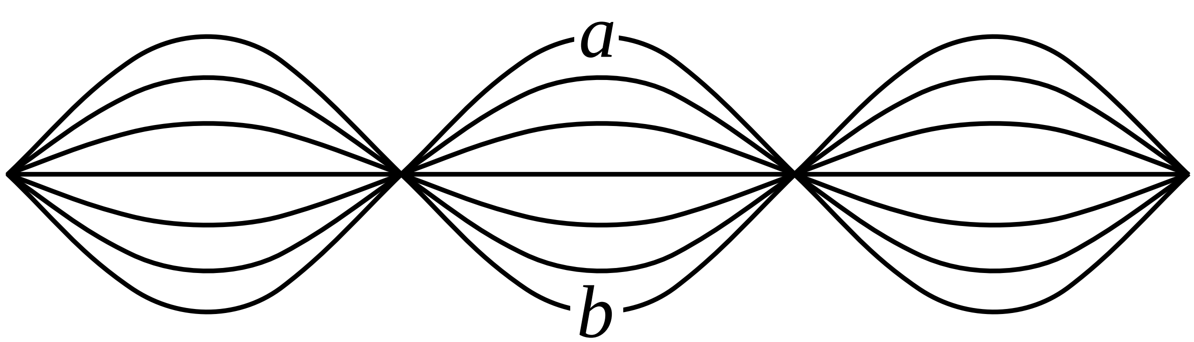
1. The second wave has a higher velocity than the first
2. The second wave has a lower velocity than the first
3. The two waves have the same velocity
4. More information is needed

Two strings, one thick and the other thin, are connected to form one long string. A sine wave travels along one string and continues through the connection onto the other string. Which of the following is the same on both strings:

1. frequency
2. wavelength
3. wave speed (i.e., propagation speed)
4. two of the above
5. all of the above
6. none of the above

Standing Waves

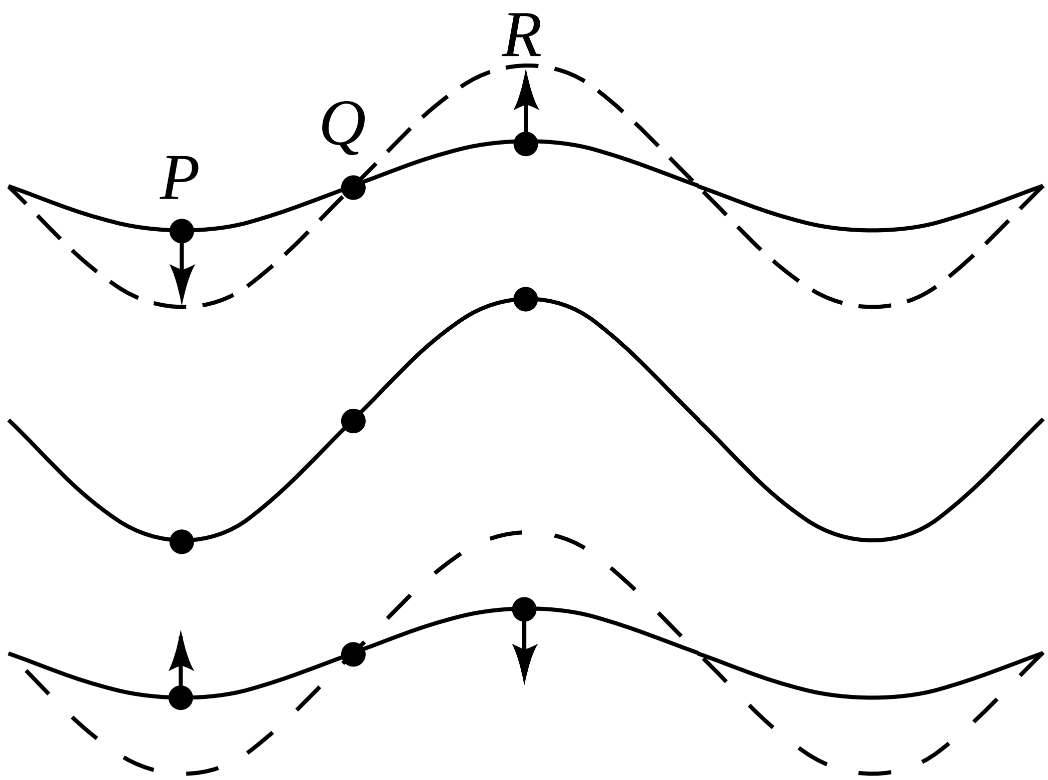
A string is clamped at both ends and plucked so it vibrates in a standing mode between two extreme positions *a* and *b*. Let upward motion correspond to positive velocities. When the string is in position *a*, the instantaneous velocity of points along the string



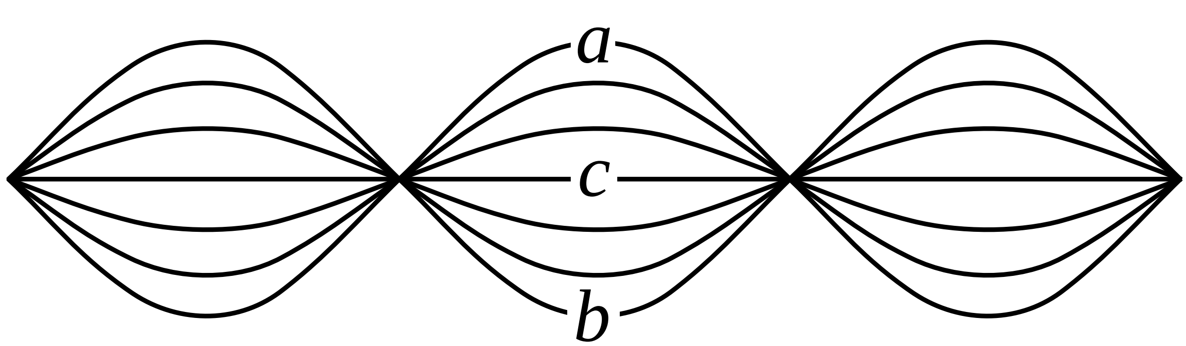
1. is zero everywhere.
2. is positive everywhere.
3. is negative everywhere.
4. depends on location.

ANS: **1**—The instantaneous velocity is zero everywhere in string position *a*.

Every point on the string undergoes simple harmonic motion in the vertical direction. At orientation *a*, every point on the string is at its maximum displacement (most positive or most negative) from equilibrium, i.e. each point on the string is at the amplitude of its simple harmonic motion, at which point its velocity is zero (each point is turning around). The figure below shows the motion of the string just before orientation *a*, just at orientation *a*, and just after orientation *a*. The same would be true for orientation *b*.



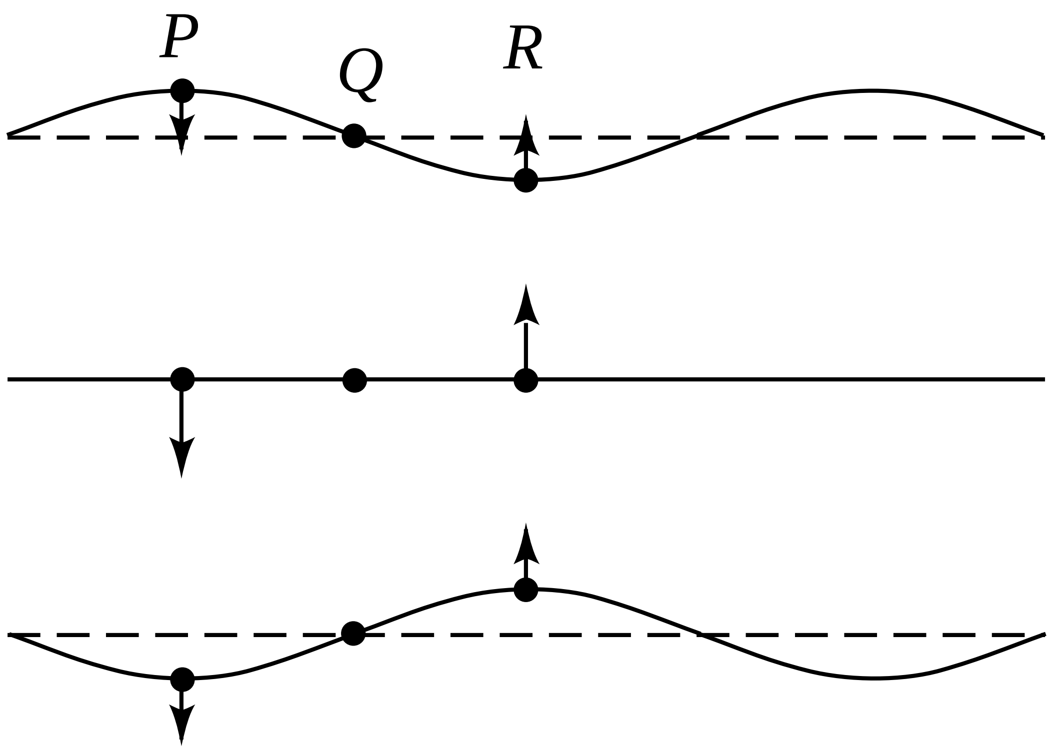
A string is clamped at both ends and plucked so it vibrates in a standing mode between two extreme positions a and b. Let upward motion correspond to positive velocities. When the string is in position c, the instantaneous velocity of points along the string



1. is zero everywhere.
2. is positive everywhere.
3. is negative everywhere.
4. depends on location.

ANS: **4**—The instantaneous velocity depends on location along the string.

At orientation *c*, each point on the string is at its equilibrium position of simple harmonic motion. This is where the speed of each point is greatest. Some points will be moving up, and the others moving down as in the diagram below.

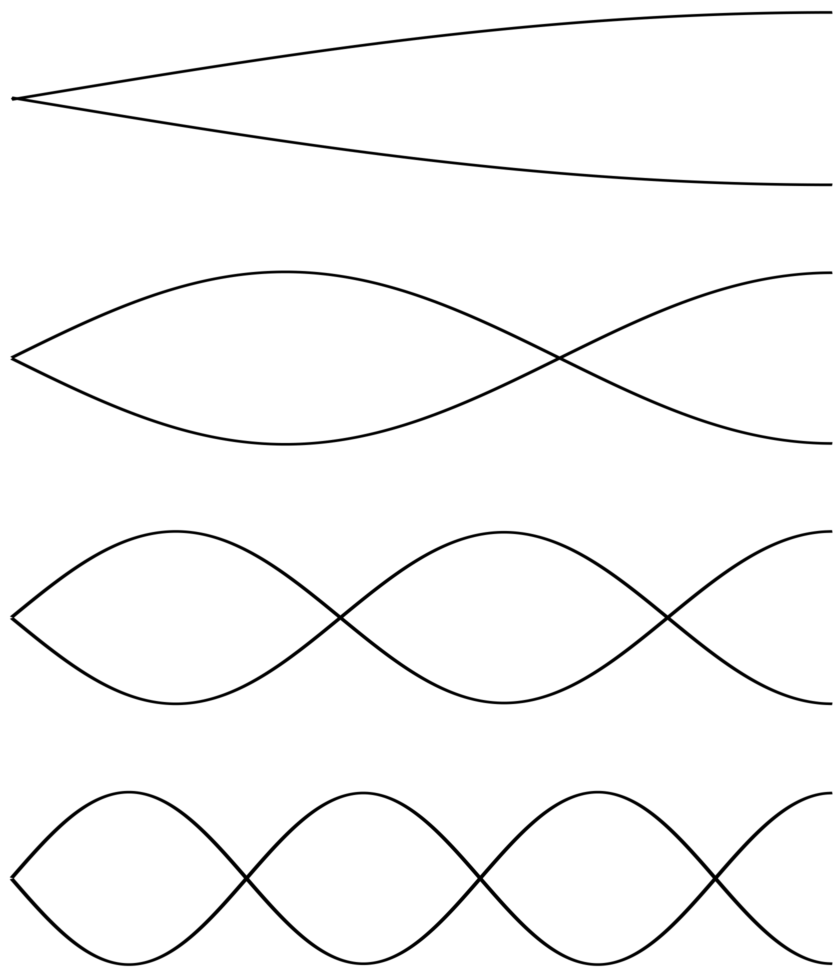


A sinusoidal standing sound wave is produced inside a tube that is closed at one end and open at the other end. What must fit between the tube’s ends?

1. An integer number of wavelengths
2. An integer number of half wavelengths
3. An odd integer number of quarter wavelengths

ANS: **3**—An odd integer number of quarter wavelengths fit in the tube.

There must be a displacement node at the closed end of the tube and a displacement antinode at the open end of the tube. There may be other nodes in-between the ends, but the standing wave pattern will include an integer number of “footballs”: an integer number of half-wavelengths or an *even integer* number of quarter wavelengths, plus a “half football” or one quarter wavelength. (See standing wave patterns below.) The allowed modes make the length of the tube equal to $\lambda/4$, $3\lambda/4$, $5\lambda/4$, etc.–an odd number of quarter wavelengths.



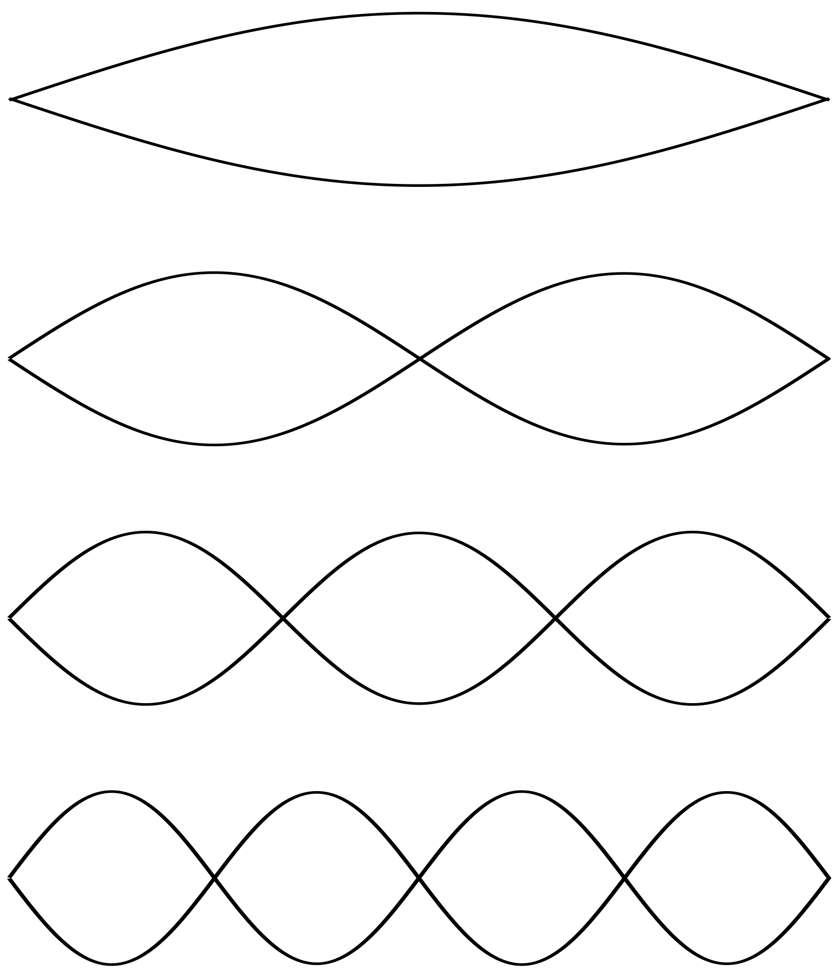
A sinusoidal standing sound wave is produced inside a tube that is open at both ends. What must fit between the tube's ends?

1. An integer number of wavelengths
2. An integer number of half wavelengths
3. An odd integer number of quarter wavelengths

ANS: **2**—An integer number of half wavelengths fit in the tube.

There must be a displacement antinode at each open end, with one or more nodes in between. Therefore, there will always be two “half footballs”, one at each end, and an integer number of footballs in-between. This means that there will be an integer number of half-wavelenghts in the standing wave pattern.

Alternatively, you can look at the problem in terms of *pressure* nodes and antinodes. Each open end of a pipe corresponds to a pressure node. Therefore, just like waves on a string clamped at both ends, there will always be a node at each end and zero or more nodes in-between. This means that there are an integer number of “footballs”, or half-wavelenghts, in the standing wave pattern. (See standing wave patterns below. These represent the pressure nodes and antinodes. You can draw new patterns, replacing all nodes with antinodes and vise versa, to see the representation in terms of displacement of air molecules.)



If the frequency of a wave in a string is increased, how does the velocity of the wave change?

1. increases
2. remains the same
3. decreases

ANS: **2**—The velocity of the wave remains the same when we increase the frequency.

The speed of a wave on a string depends on mechanical properties of the string, such as tension and mass density (thickness). Changing the frequency of the wave does not change these properties, so the speed remains constant. When you change the frequency and keep the speed the same, the result is to change the wavelength. This is the opposite of the way a guitar works. By holding down the string at different points on the neck, you change the wavelength, keeping the speed the same, which causes a change in frequency.

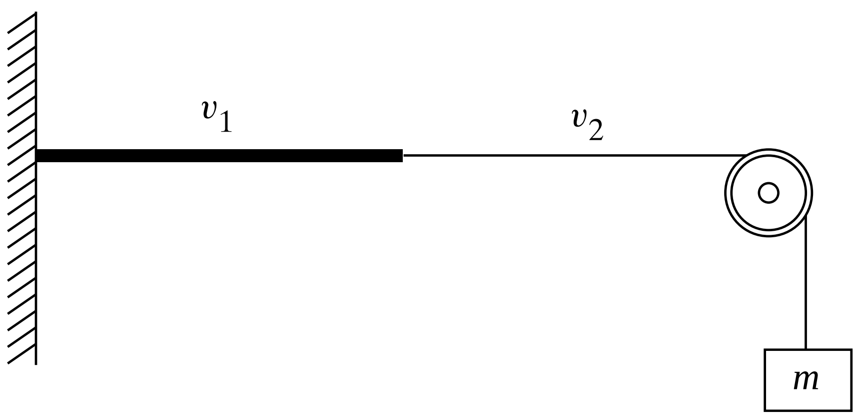
Two strings, one thick and the other thin, are connected to form one long string. A wave travels along the string and passes the point where the two strings are connected. Which of the following change(s) at that point:

1. frequency
2. period
3. propagation speed
4. wavelength
5. 3 & 4
6. 1, 2, & 4
7. All of the above
8. None of the above

ANS: **5**—The speed and wavelength change.

The strings are connected, so they have the same tension. This means that the thinner string (lower density) will have a greater wave speed. Because they are connected, each time one string moves up or down at the interface, the other will move up or down at the same time. This means that the frequency and period will be the same. However, because the wave speeds are different, the wavelengths will be different. The wavelength will be shorter in the thicker string and longer in the thinner string.

A weight is hung over a pulley and attached to a string composed of two parts, each made of the same material but one having four times the diameter of the other. The string is plucked so that a pulse moves along it, moving at speed v_1 in the thick part and at speed v_2 in the thin part. What is v_1/v_2?



1. 1
2. 2
3. 4
4. 1/2
5. 1/4

ANS: **5**—$v_1/v_2 = 1/4$

The wave speed in a string is $v = \sqrt{F/\mu}$, where F is the tension in the string, and $\mu = m/L$ is the linear mass density, or mass per unit length. A string of length L and cross-sectional area A (diameter d) is $V = AL = (\pi/4)d^2L$. Therefore, the mass of the string is $m = \rho V = (\pi/4)d^2L\rho$, were ρ is the density of the string. This makes the linear mass density of the string $\mu = m/L = (\pi/4)d^2\rho$, proportional to the diameter of the string squared.

In this problem, string 1 is four times the diameter of string 2, so it has 16 times the linear mass density, $\mu_1 = 16\mu_2$. This means that $v_1 = \sqrt{F/\mu 1} = \sqrt{F/16\mu_2} = (1/4)\sqrt{F/\mu_2} = v_2/4$.

Warmup Question

Stringed instruments like guitars and violins are designed to change the frequency of vibrations of the string when you hold the string at different points along the neck. When you hold the string down somewhere on the neck, you shorten the effective length of the string compared to if you were not pinching the string. Explain how this affects the frequency of the note played and whether holding the string down somewhere on the neck will create a higher-frequency or lower-frequency note compared to the unheld string.

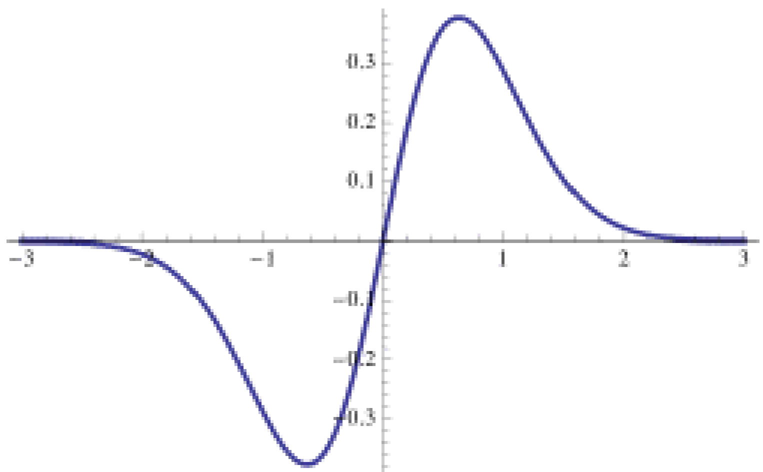
ANS: By decreasing the length of the string, you decrease the wavelengths of the harmonics. Because the wave speed doesn’t change (it depends on the tension and thickness of the sting, not its length), the frequencies will increase as the wavelengths decrease.

Warmup Question

A wave pulse in one dimension is described by

$$f(x,t) = \exp\left[(x+t)^2\right] \operatorname{arccot}(1/(x+t)^2)\,,$$

which may be an intimidating equation but at t = 0 simply looks like this as a function of x:



The peak value of $f(x,t)$ is 0.3780, and in this picture, at $t = 0$, that occurs at $x = 0.6328$. At a later time $t = 1$, at what value of x does $f(x,t)$ take that same value of 0.3780? What about at $t = 2$? What does that tell you about the motion of this pulse? Explain your reasoning. (Hint: this one is not hard if you think first and don't immediately start trying to crank out tons of algebra.)

ANS: For the peak in question, at $t = 0$, $x_{\text{peak}} = 0.6328$, so $x_{\text{peak}} + t = 0.6328$. The peak will *always* be located at (x, t) such that $x_{\text{peak}} + t = 0.6328$. This means we can determine the location of the peak at any time by computing $x_{\text{peak}} = 0.6328 - t$. At $t = 1$, the peak is located at $x_{\text{peak}} = 0.6328 - 1 = -0.3672$, while at $t = 2$, the peak is located at $x_{\text{peak}} = 0.6328 - 2 = -1.3672$.

From this result we can see that the wave is moving to the left. In fact, it is moving to the left with speed $v = 1$. The lines $x + t = \text{const.}$ are called *characteristics*. If you know the value of the waveform at any given (x, t), you can find the location of that point on the wave by computing $x = \text{const.} - t$.

Warmup Question

For a typical trampoline, what condition would you expect to hold around the edge where the elastic sheet attaches to the frame? At the edge there should be a displacement

1. mode
2. node
3. antimode
4. antinode

ANS: **2**—There will be a node all along the edge.

The sheet is not allowed to move at the edge. This makes the entire edge a displacement node. (At a node, there is no change in the quantity in question.)