

A refrigerator works by absorbing heat from one region (the interior) and expelling it into another (typically behind it, where those dusty coils are). The trick is to expand a gas that is confined in a series of coils in contact with the cold interior so that it absorbs heat. That gas is then transferred to another series of coils that are in contact with the hot exterior where it is compressed so that it gives off heat.

Let's say you have an ideal refrigerator (whose engine works without friction) but a non-ideal roommate (who never works without coercion) who regularly leaves the door ajar. Hence the distinction between "interior" and "exterior" is lost as air circulates freely between them. Can your refrigerator be good enough that it expels exactly the same amount of heat as it absorbs? Make your argument using the first law of thermodynamics.

PV diagrams are your friend

A refrigerator goes through four processes to make its interior cooler than its exterior. Starting at some value for  $P$  and  $V$ , it does the following: 1) Isovolumetric cooling, 2) isothermal expansion, 3) isovolumetric heating, and 4) isothermal compression. This allows heat to be taken in when in the cold environment and expelled when in the hot environment. The PV diagram describing this cycle looks like a downward sloping region bound on the left and right by vertical lines. This cycle goes counterclockwise. Thus, the gas does negative work at a higher pressure than when it does positive work. As a result, the net work must be negative. Now, as  $\Delta U = Q - W = 0$ , then  $Q = W$ . So, as  $W$  is negative,  $Q$  must also be negative. This means that the refrigerator is always expelling more heat than it is taking in. So, it will never expel the exact same amount of heat that it absorbs. It will always make the room hotter in the long run if it is open. The only way that it would expel the exact same amount of heat that it absorbs is if the heat reservoirs that it uses do the two isothermal processes become the same temperature. In other words, the only way that the refrigerator will expel the same amount of heat that it absorbs is if it is kept running long enough that the two vertical lines on the PV diagram shrink into non-existence. This would make  $W = 0$ , and thus  $Q = 0$ .

Another good example, just some subtle wording fixes

The whole idea of a heat engine is that through a given cycle that has the same starting and endpoint on a PV diagram, there is no net energy, but there is also work done. So, by the first law of thermodynamics ( $\Delta U = \Delta Q - W$ ), there must be a heat absorbed or emitted. For a refrigerator to be good enough to expel the same amount of heat that it absorbs,  $\Delta Q$  would be equal to zero. However, work would still have to be done for the refrigerator to function properly and expel heat necessary to keep the temperature inside cool. This conditions would make the change in internal energy non zero, which is not a characteristic of a heat engine. Thus, the refrigerator cannot be good enough to achieve this.