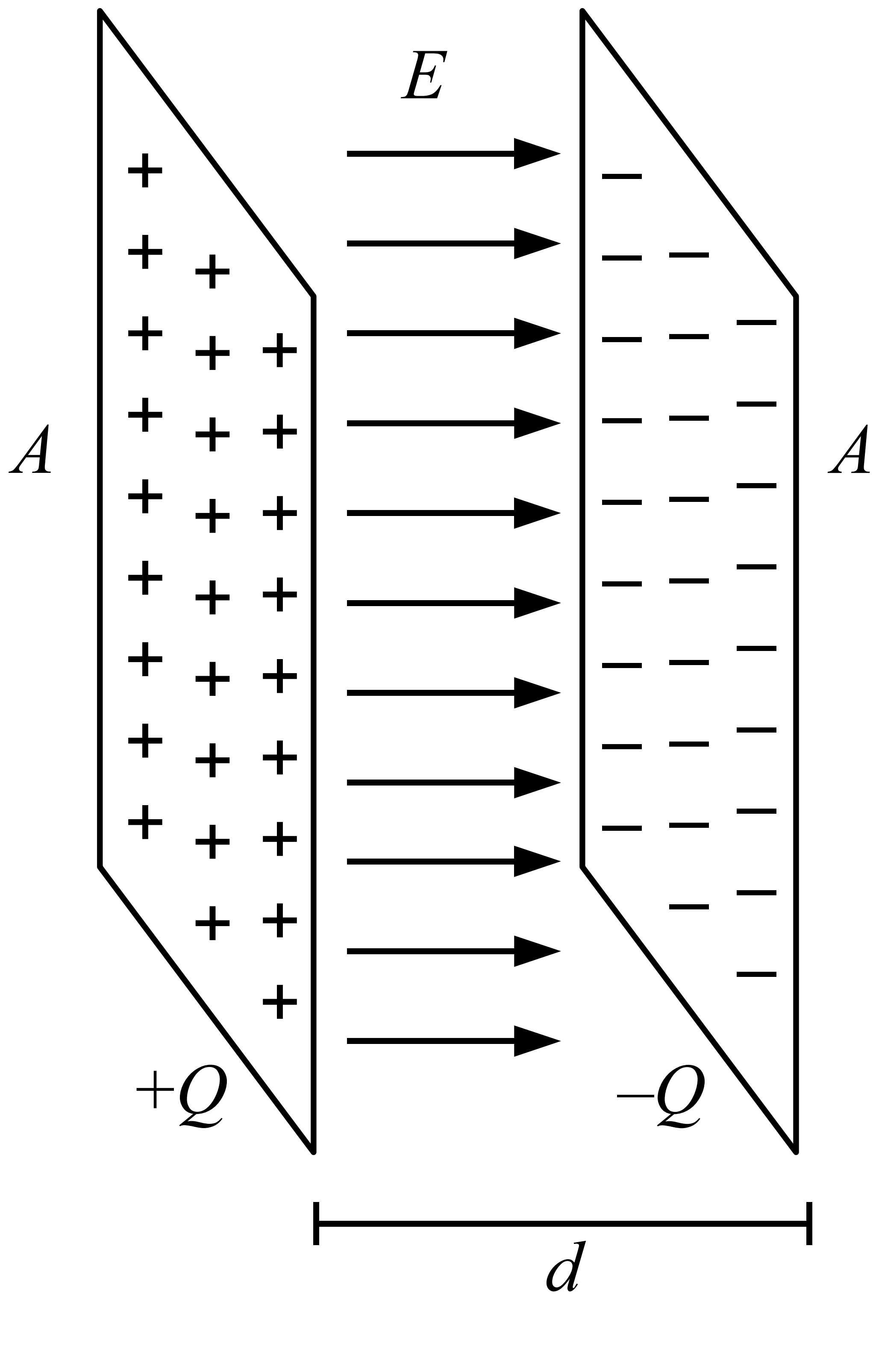
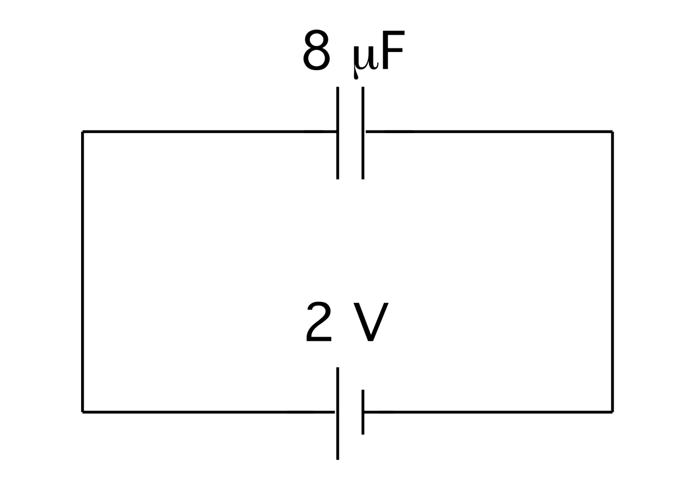
Capacitance and Capacitors



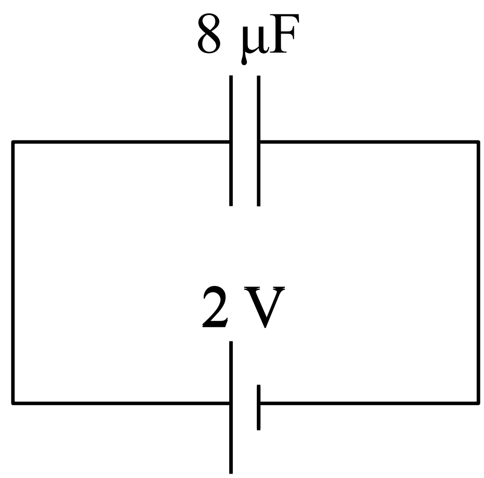
A 2 Volt power supply is connected to a 8 μF capacitor as shown below.



What is the magnitude of the current passing through the capacitor?

1. zero
2. 0.25 μC
3. 4.0 μC
4. 8.0 μC
5. 16.0 μC
6. need more information

A 2 V power supply is connected to a 8 μF capacitor as shown. What is the magnitude of the charge stored on the capacitor?

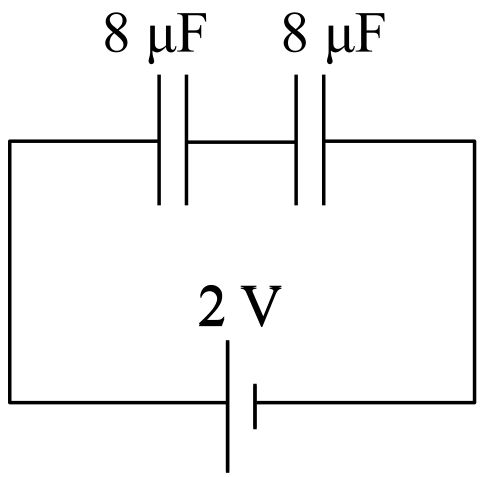


1. zero
2. 0.25 μC
3. 4.0 μC
4. 8.0 μC
5. 16.0 μC
6. need more information

ANS: **5**— $Q = 16.0\,\mu\text{C}$

The potential difference across the $8\,\mu\text{F}$ capacitor is $2\,\text{V}$, so the charge on the capacitor is $Q = C\Delta V = (8\,\mu\text{F})(2\,\text{V}) = 16\,\mu\text{C}$.

A 2 V power supply is connected to a pair of 8 μF capacitors as shown. What is the magnitude of the charge stored on the first capacitor?



1. zero
2. 0.25 μC
3. 4.0 μC
4. 8.0 μC
5. 16.0 μC
6. need more information

ANS: **4**— $Q = 8.0\,\mu\text{C}$

Because the two capacitors are identical, each will have a potential difference of 1 V to give a total of 2 V potential difference across the pair. The charge on each capacitor, therefore, will be

$$Q = C\Delta V = (8\,\mu\text{F})(1\,\text{V}) = 8\,\mu\text{C}\,.$$

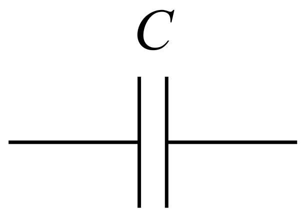
The above argument only works because the capacitors are identical. If they were not identical, they would not have the same potential difference, although the total potential difference across the pair will still be 2 V. Here I will give an alternative argument that will always work, even if the capacitors are not equal.

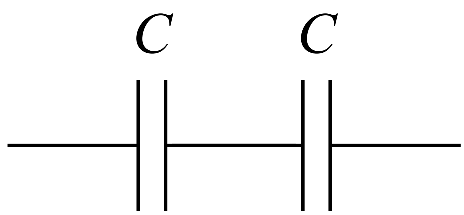
First of all, these capacitors are in series. When connected to the battery, they will receive the same charge. We can replace the pair with a single "equivalent" capacitor that will draw the same charge from the battery. The equivalent capacitance for a series pair is found from the formula

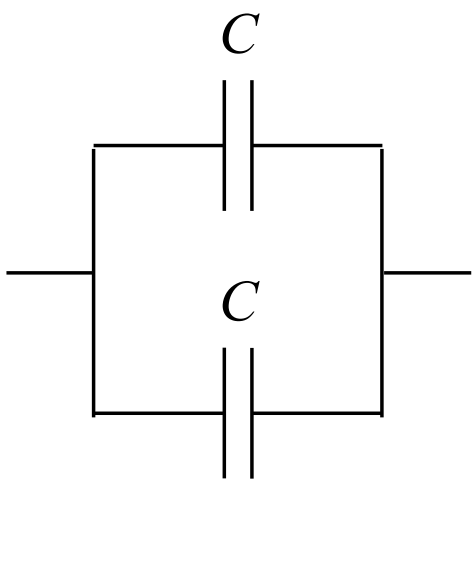
$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \longrightarrow \quad C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2}\,.$$

In this case, the equivalent capacitance of the pair is $4\mu\text{F}$. This gives a charge of $Q = C\Delta V = (4\,\mu\text{F})(2\,\text{V}) = 8\mu\text{C}$ across the equivalent capacitor, and hence across each capacitor.

Which configuration below has the greatest equivalent capacitance?

1. 

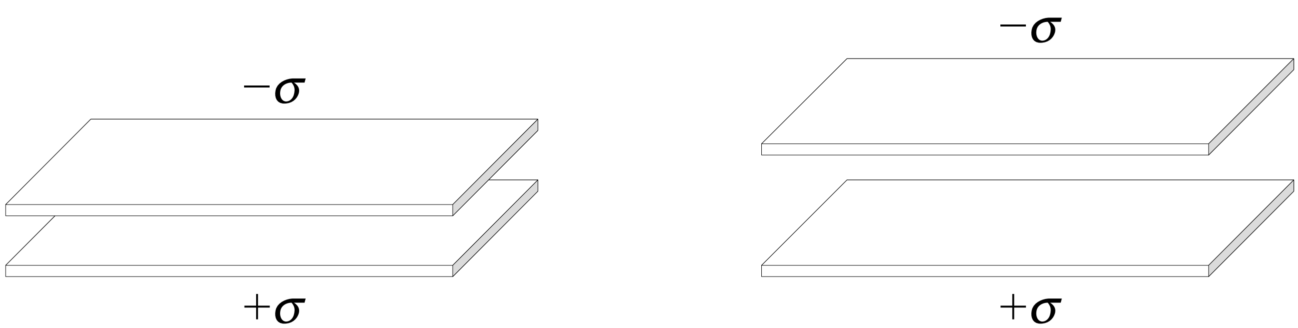
2. 

3. 

ANS: Configuration **3** has the highest equivalent capacitance.

Remember that capacitances add in parallel, meaning that additional capacitors in parallel increase the equivalent capacitance. The equivalent capacitance of #3 is $2C$. Capacitances add reciprocally in series, meaning that additional capacitors in series reduce the equivalent capacitance. The equivalent capacitance for #2 is $C/2$.

Consider a pair of parallel-plate capacitors with identical plate areas and identical charge density on them. The distance between the plates is greater for one than for the other.



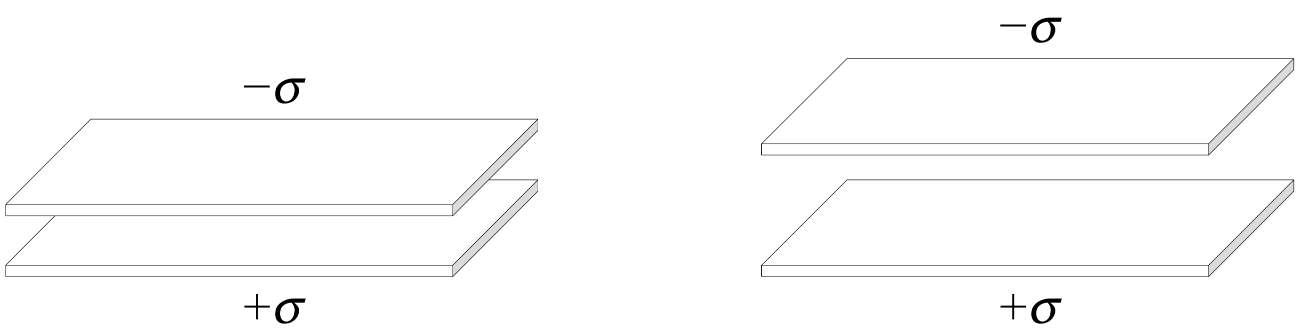
Which capacitor has the stronger electric field between the plates?

1. the one with closer plates
2. the one with more separated plates
3. the field is the same
4. more information needed

ANS: **3**—The field is the same in both cases.

As long as the plates are closer together than the plate size, the approximation we always use for parallel-plate capacitors, the field between the plates only depends on the charge density, or equivalently the ratio of charge to plate area. These quantities are the same for both capacitors.

Consider a pair of parallel-plate capacitors with identical plate areas and identical charge density on them. The distance between the plates is greater for one than for the other.



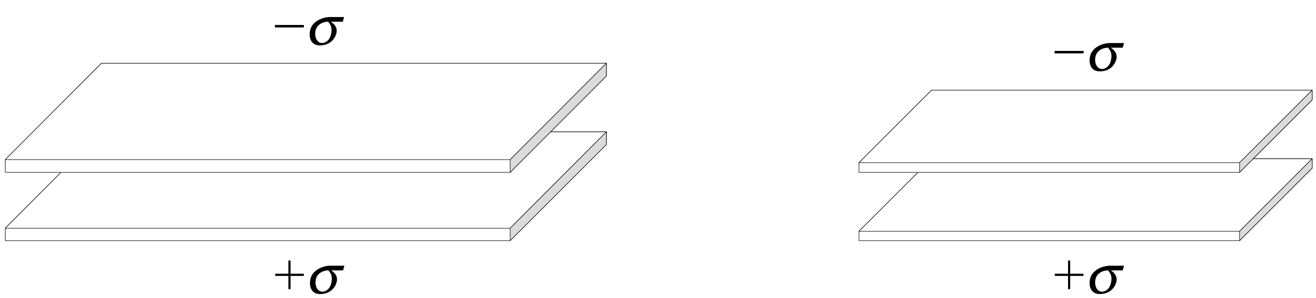
Which capacitor has the higher potential difference between its plates?

1. the one with closer plates
2. the one with more separated plates
3. the potential differences are the same
4. more information needed

ANS: **2**—The one with the more separated plates has the greater potential difference.

Because the field between the plates is uniform, the potential difference is just the product of the field and the plate separation, *d*. The field is the same for both capacitors, so the potential difference will be greater for the capacitor with the larger plate separation.

Consider a pair of parallel-plate capacitors with identical spacing between the plates and identical charge density on them.



The area of the plates is greater for one than for the other. Which capacitor has the stronger electric field between its plates?

1. the one with bigger plates
2. the one with smaller plates
3. the field is the same
4. more information needed

ANS: **3**—The field is the same in both cases.

The field between the plates only depends on the charge density, which is the same for both capacitors. The one with the larger plate area has a larger charge, but the charge densities and hence the fields between the plates are the same for both capacitors.

Note: The three previous two questions help establish a formula for the capacitance of a parallel plate capacitor. Capacitance is defined to be the ratio of the charge stored in a capacitor to its potential difference, $C = Q/\Delta V$. The second question above establishes that, for the same charge, the potential difference is proportional to the plate separation. This means that the capacitance will be inversely proportional to the plate separation. The third question above establishes that to keep the potential difference constant on a capacitor with increasing plate area, you must increase the charge proportionally to keep the charge density constant. In other words, the capacitance is proportional to the plate area. These observations tell us that, up to a constant, the capacitance of parallel plates is proportional to the area and inversely proportional to the plate separation: $C = \epsilon_0 A/d$.

The parallel plates of a charged capacitor are pulled apart (without changing the charge on the capacitor). Does the potential difference between the plates

1. increase
2. decrease
3. stay the same
4. more information needed

ANS: **1**—The potential difference between the plates increases.

The charge and area are fixed, so the charge density and hence the field between the plates will not change. Increasing the spacing in a constant field increases the potential difference between the plates. To keep the charge constant, as required by the question, the capacitor must not be connected to any battery or other charge source/sink.

The parallel plates of a charged capacitor are pulled apart (without changing the charge on the capacitor). Does the energy stored in the capacitor

1. increase
2. decrease
3. stay the same
4. more information needed

ANS: **1**—The energy stored in the capacitor will increase.

We must do work to pull apart the two oppositely-charged plates (which are attracted to each other), meaning that we put energy into the capacitor through work. The above reasoning requires no math or memorization of formulas. However, you should keep in mind that the energy stored in a capacitor is expressed by the following equivalent formulas:

$$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C} .$$

For this problem we are keeping the charge fixed, and we know that we are increasing the potential difference, so we can use the first expression to conclude that the energy must also increase. Alternatively, noting that the charge is fixed and the capacitance decreases, you can use the last expression to make the same conclusion. While reasoning with formulas has an appeal, please try to understand how qualitatively understanding work can lead you to the same conclusion.

The parallel plates of a charged capacitor are connected to a power supply that establishes a constant potential difference between the plates. If we pull apart the plates of the capacitor (without changing the potential difference between the plates), does the energy stored in the capacitor

1. increase
2. decrease
3. stay the same
4. more information needed

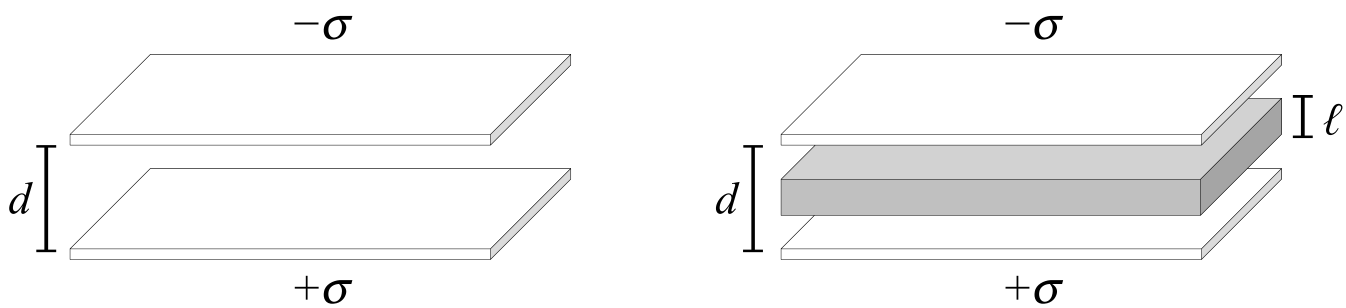
ANS: **2**—The energy stored in the plates decreases.

We know that the potential difference is fixed and the capacitance decreases as we separate the plates. Therefore, using the second energy expression above (involving C and V only) we see that the energy decreases. Apparently we do negative work on the capacitor (the capacitor does work on its surroundings) when we pull the plates apart. How can this be since the plates are still oppositely charged and attracted to each other? What is different from the previous example?

Because the capacitor is connected to a battery, charge is free to leave the capacitor and return to the battery. Separating the plates reduces the capacitance and therefore the charge stored on the capacitor maintained at the fixed voltage. Charges stored on each capacitor plate have the same sign and experience a repulsive force. The potential energy decreases as we reduce the capacitance and return some of the charge to the battery. This is not the case in the previous example, where all of the charges remain on the plates.

You can think of it this way: a battery supplies energy to a capacitor when it charges it. When you reduce the capacitance, some of the charge returns to the battery, taking energy with it.

Consider a capacitor made of two parallel metallic plates separated by a distance d. The top plate has a surface charge density $-\sigma$, the bottom plate $+\sigma$.



A slab of metal of thickness $\ell < d$ is inserted between the plates, not connected to either one. Upon insertion of the metal slab, the potential difference between the plates

1. increases
2. decreases
3. stays the same
4. more information needed

ANS: **2**—The potential difference between the plates decreases.

There is no electric field inside the metal slab because it is a conductor. There is a uniform electric field in the air gaps between the plates and the slab. The potential difference between the plates is just the product of the field and the distance over which there is a field, $\Delta V = E(d - \ell)$.