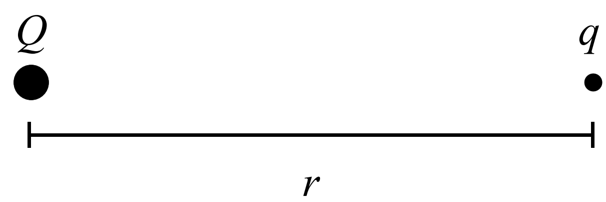
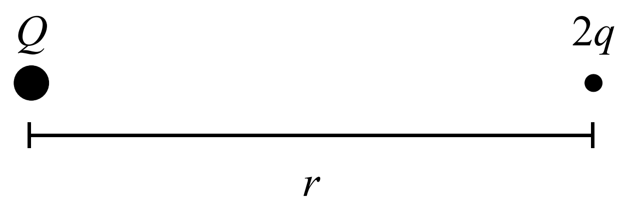
Potential and Energy

A positive charge Q is a rest at the origin. First, a positive test charge q is brought to a distance r from Q from very far away.



Then q is removed and another positive test charge $2q$ is brought to the same position.



For which test charge is the electric potential energy greater?

1. q
2. $2q$
3. same for both
4. need more information

ANS: **2**—The electric potential energy is greatest at the position for charge 2*q*.

Electrical potential energy is a measure of how much work is required to move the charge from very far away to the point under consideration. The field at all points in space is created by charge *Q*, and each test charge “feels” a force proportional to this field. The charge 2*q* will have twice the electrical force on it as the charge *q* at any given point, so it takes more work to move 2*q* to the final position. Therefore, the potential energy for 2*q* is greater than the potential energy for *q*. In fact, it will have twice as much potential energy. (You could also just rely on the fact that potential energy depends on the product of the charges, but it gives us better physical insight to think in terms of work done moving a charge against a force.)

Please note in this and the following questions that I used the phrase “potential energy for a test charge” as shorthand. In reality, the potential energy is stored in the interaction between the two charges. It does not “belong” to one or the other.

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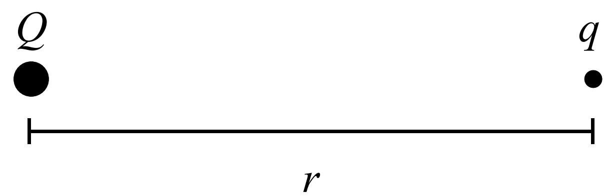

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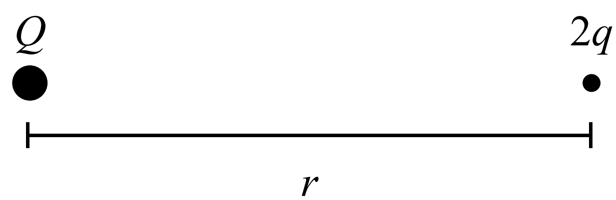


For which test charge is the electric potential at that point greater?

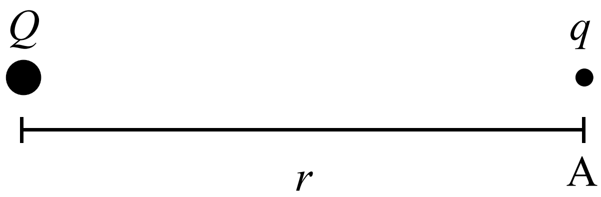
1. q
2. $2q$
3. same for both
4. need more information

ANS: **3**—The electric potential is the same in both cases.

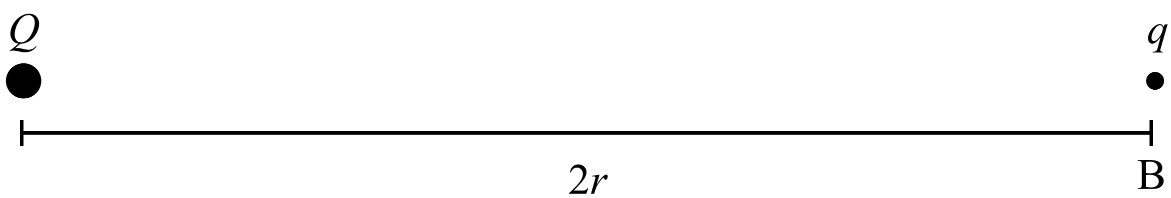
The electric potential due to charge Q exists at the point in space where the test charges are located, even if those test charges were not there! This is analogous to the field, which is created by one charge, exists at all points in space, and causes a force on any test charge that might be located at some point.

In other words, the electric potential V is the potential energy of a test charge divided by that charge. The charge $2q$ has twice the potential energy, but when dividing by charge we find that the potential at the point in question is the same regardless of what test charge is there. (Please note carefully the phrasing that asks about the electrical potential at the "test charge position," not the potential on the test charge.)

Two test charges are brought separately into the vicinity of a positive charge Q. First, a test charge $+q$ is brought to point A, a distance r from $+Q$.



Next, $+q$ is removed and a test charge $+q$ is brought to point B, a distance $2r$ from $+Q$.



Compared with the electric potential at A, the electric potential at B is:

1. greater.
2. smaller.
3. the same.

ANS: **2**—The electric potential at point B is smaller than at point A.

The electric potentials at A and B do not depend on the charges at those points, but they do depend on the distance from Q. Electric potential decreases with distance, so the potential at B should be smaller than the potential at A. Specifically, because it is twice the distance from charge Q, the potential at B should be *half* the potential at A.

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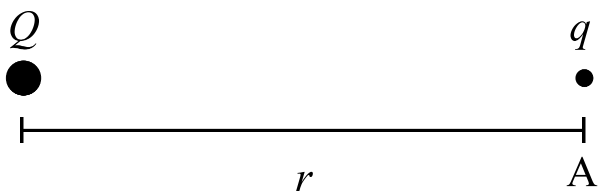
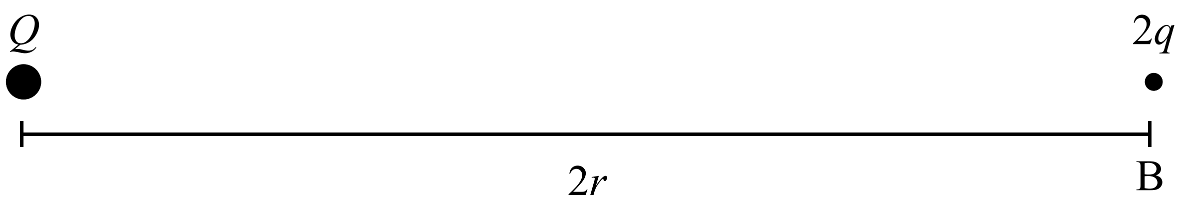


Next, $+q$ is removed and a test charge $+2q$ is brought to point B, a distance $2r$ from $+Q$.



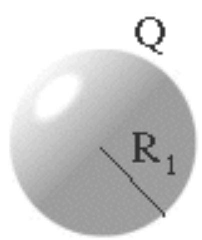
Compared with the electric potential at A, the electric potential at B is:

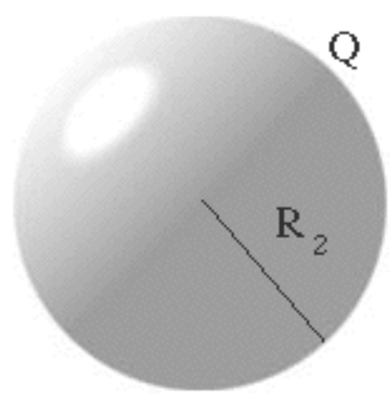
1. greater.
2. smaller.
3. the same.

ANS: **2**—The electric potential at B is smaller than the electric potential at B.

Again, the electric potentials at A and B do not depend on the charges there. The potential at B should be *half* the potential at A.

Consider two spheres of different radii. On the surface of each, the same quantity of charge Q is distributed uniformly. Examining each sphere by itself (with the other sphere absent), we measure the electric potential relative to infinity at a point some fixed distance *r* away from the center of that sphere. Which measurement would give the higher potential at that point?





1. The larger sphere
2. The smaller sphere
3. The potentials would be the same
4. Need more information

Consider two spheres of different radii. On the surface of each, the same quantity of charge Q is distributed uniformly. Examining each sphere by itself (with the other sphere absent), we measure the electric potential relative to infinity at its surface. Which measurement would give the higher potential?

1. The larger sphere
2. The smaller sphere
3. The potentials would be the same
4. Need more information

A positive test charge will accelerate toward regions of ______ electric potential and ______ electric potential energy.

1. higher; higher
2. higher; lower
3. lower; higher
4. lower; lower

ANS: **4**—A positive test charge will accelerate toward regions of *lower* potential energy and *lower* gravitational potential energy.

The electric *force* on a charge points in the direction of decreasing potential *energy*, so all charges will accelerate toward regions of lower potential energy. The electric *field* points in the direction of decreasing *potential*. For a positive charge, the force points in the direction of the field, so positive charges will accelerate toward regions of lower potential.

A negative test charge will accelerate toward regions of ______ electric potential and ______ electric potential energy.

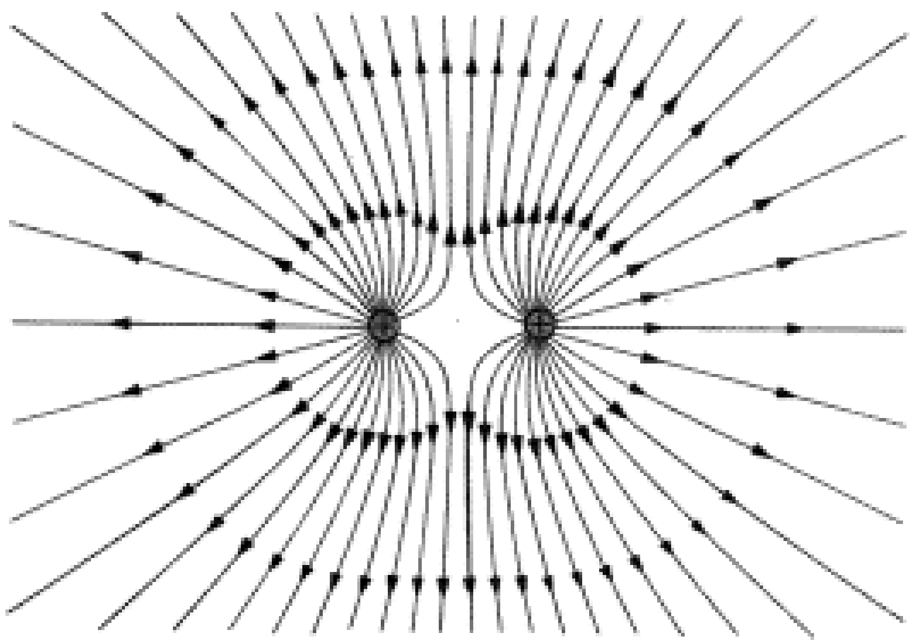
1. higher; higher
2. higher; lower
3. lower; higher
4. lower; lower

ANS: **2**—A negative test charge will accelerate toward regions of *higher* electrical potential and *lower* electrical potential energy.

Again, the electric *force* on a charge points in the direction of decreasing potential *energy*, so all charges will accelerate toward regions of lower potential energy. The electric *field* points in the direction of decreasing *potential*. For a negative charge, the force points opposite the direction of the field, so negative charges will accelerate toward regions of higher potential.

Warmup Question

The figure below shows the electric field lines for a matched pair of positive charges. Exactly halfway between them, the field is zero because the fields they produce individually are of equal magnitude but point in opposite directions. What about the electric potential (defined such that $V = 0$ at infinity) at that same point? Please discuss qualitatively without relying on equations to support your conclusions.



ANS: The electric potential will not be zero. Recall that the potential at a point is equal to the sum of the potentials at that point due to all charges. Here both charges are positive and each will produce a positive potential at the center. The total potential at that point, therefore, is positive.

However, the *electric field* is zero at the center point. Since the electric field is a measure of the change in potential with position (a derivative), this implies that the potential is a local maximum or minimum at the center (and is effectively constant within a small region around the center).

There is another way to see that the potential at the center will not be zero. If you consider the field lines, you know they have to bend away like that, because field lines always point away from positive charges. Near each charge, the field must point away from the charge. However, they can't point away from both charges at the center, by symmetry. Instead, the field lines will curve away from the positive charges as shown in the figure above. At great distances, they point away from both positive charges.

Once you agree that the equipotentials and field lines shoudl look like they do, now consider the picture in light of the fact that field lines always point in the direction of decreasing potential. Near the charges we have field lines that point toward the central point between them, so we know that this central point must have a lower potential than near the charges. However, as we continue to follow the field lines, they bend away from the center, and toward infinity, again in the direction of decreasing potential. Therefore, the central point must have a greater potential than infinity, which has zero potential.

Warmup Question

A light bulb works by moving charges through an electric potential. Those charges gain kinetic energy, which is converted into heat and light. For an ordinary light bulb that operates at 100 Watts ($= 100$ J/s), estimate the amount of charge that passes through it in one second. (Typical household wiring operates at 110 Volts, which for purposes of estimation is, of course, ≈ 100 V, right?)

ANS: The charges lose a potential energy of 100 J every second as they pass through the bulb. With a potential difference of 100 V ($= 100$ J/C) we see that 1 coulomb of charge must pass through the bulb every second. Hey, that gives you a rough idea of how much charge a coulomb represents, right?

Warmup Question

Where is the most convenient place to set electric potential equal to zero when dealing with a single point charge?

1. At that charge’s location
2. At the location of test charge you use to probe its influence
3. A standard reference distance 1 meter from the charge
4. Infinitely far away

ANS: **4**—It is typically convenient to set electric potential to be zero infinitely far away from the charge. This gives a very convenient formula for potential in terms of the distance from the charge, $V = k_e q/r$.