Capacitance

A solid spherical conductor is given a net nonzero charge. The electric potential of the conductor is

1. largest at the center.
2. largest on the surface.
3. largest somewhere between center and surface.
4. constant throughout the volume.
5. We need to know the sign of the charge before answering

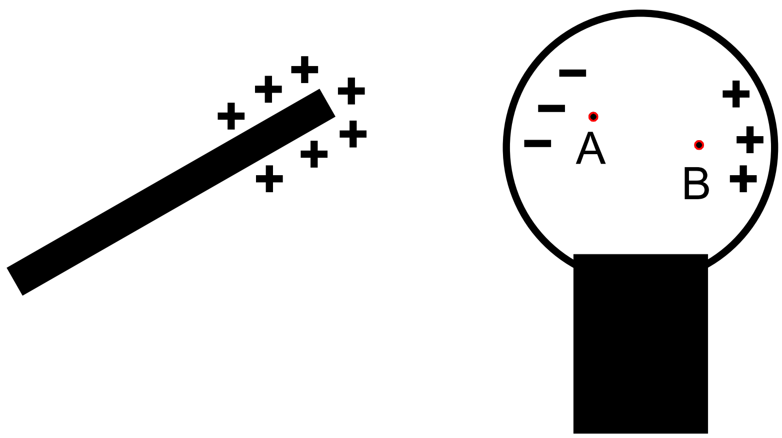
ANS: **4**—The electric potential is constant throughout the volume.

You learned this in your laboratory activity last week. The potential is everywhere constant on a conductor at least in the *static* case. Why is that?

A conductor is a material in which certain “conduction” electrons are free to move. Suppose this were not the case and that there was a non-zero electric field in the conductor. Then there would be a force on the conduction electrons, causing them to move around. Eventually charges will move to the edge of the cylinder. Will be field be zero now? If not, there will be a net force on conduction electrons and the process continues until a truly static case is reached.

The electric field indicates change in electric potential. The absence of a field indicates a constant electric potential.

A positively charged rod is held near a neutral conducting sphere as illustrated below. The sphere polarizes as shown and the electric field settles into a static configuration. A test particle with a small positive charge is now moved from point A to point B. The mechanical work required to cause this motion



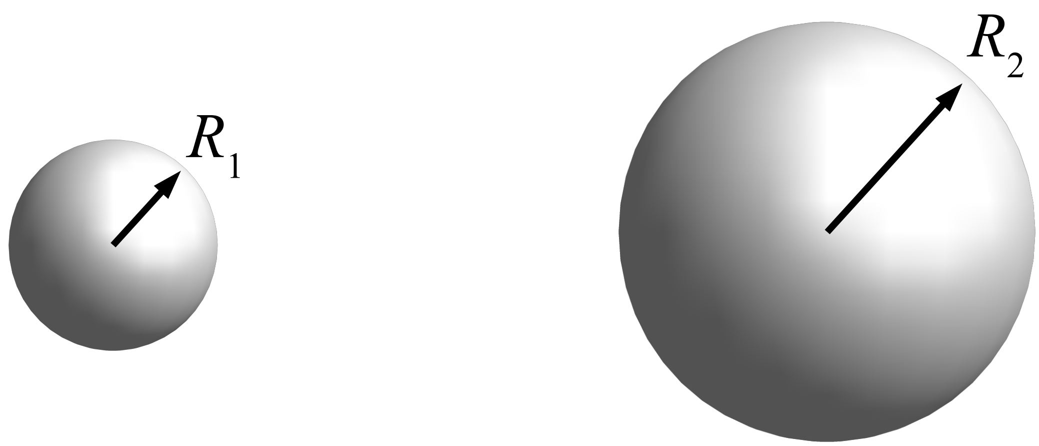
1. is positive.
2. is zero.
3. is negative.
4. depends on the path taken from A to B.
5. cannot be determined without knowing more about the polarization induced in the sphere.

ANS: **2**—No work is required to move the charge.

There is no electric field within a conductor in steady state. Even though there are excess charges on the surface of the sphere, their contribution to the total field simply cancels the field that the rod would produce inside the sphere. That is, the induced electric field cancels the applied field everywhere inside the conductor. So long as the charge we're moving about is small (so it doesn't disturb the other charges significantly), it will feel no electric force. Therefore, it takes no work to move it about.

It may even be easier to understand in light of the answer to the previous question. A conductor in the steady state is an equipotential. In particular, $\Delta V(A \to B) = 0$. That means that, for a test charge q, the change in potential energy is $\Delta U(A \to B) = q\Delta V(A \to B) = -W_{\mathrm{C}} = 0$. No work is done by the conservative electric force, so no work is required to move the charge.

Consider two spheres of different radii. On the surface of each, the same quantity of positive charge Q is distributed uniformly. For each sphere, we measure the electric potential (relative to infinity) at a point some given distance r (where $r > R_2 > R_1$) away from the center of that sphere.



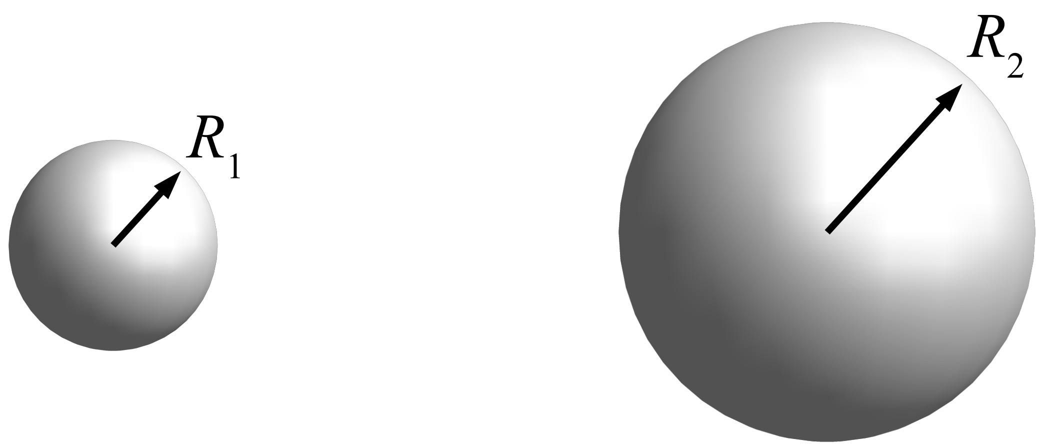
Which measurement would give the higher potential at that point?

1. The larger sphere
2. The smaller sphere
3. The potentials would be the same
4. Need more information

ANS: **3**—The potential at r will be the same for either sphere.

Outside the sphere, the electric fields from both are identical to that of a point charge of magnitude Q. Hence, pushing a test charge around takes exactly the same effort. In particular, it takes the same amount of work to move a charge from ∞ to r, regardless of the size of each sphere.

Consider two spheres of different radii. On the surface of each, the same quantity of positive charge Q is distributed uniformly. For each sphere, we measure the electric potential (relative to infinity) *at its surface*.



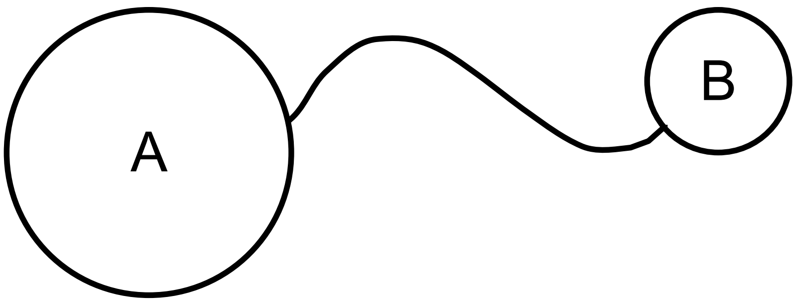
Which measurement would give the higher potential?

1. The larger sphere
2. The smaller sphere
3. The potentials would be the same
4. Need more information

ANS: **2**—The smaller sphere will have the greater potential at the surface.

As in the previous problem, the potential at a distance R_2 from the center of each sphere will be the same. However, because $R_1 < R_2$, it takes work to continue moving a test charge from a distance R_2 to the surface R_1 for the smaller sphere, indicating that the potential at the surface of the smaller sphere is greater than the potential at the surface of the larger sphere.

Consider the pair of charged metal spheres below, which are connected by a conducting wire as shown. The radius of sphere A is larger than that of sphere B.



Which of the following quantities *must* be the same for both spheres?

1. potential at the surface
2. charge on the sphere
3. surface charge density
4. field at the surface
5. more than one of the above
6. None of the above

ANS: **1**—Only the potential at the surface is the same for both spheres.

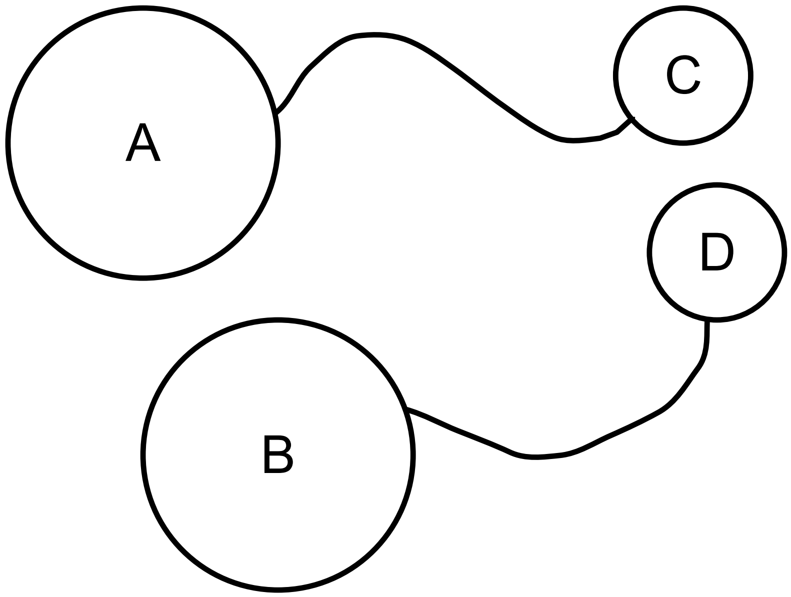
The details of the charge distribution and electric field are complicated and depend upon the details of the geometry. The only simple truth is this: in an electrostatic situation, conductors connected by a wire form a single conductor and therefore are at the same potential.

For a single sphere with excess charge on it, the surface charge density is indeed uniform. However, these two spheres will influence each other via their fields. An excess of charge will accumulate on the sides away from each other, just as if you brought a charged rod up next to a simple conducting sphere. The only way we can guarantee uniform surface charge densities is to make the wire long enough, separating the spheres far enough, that the fields created by one sphere are too weak to influence the charge distribution on the other sphere.

For the sake of argument, we've make the charge distributions uniform on the two spheres. Even then, the fact that their potentials are the same implies that their total charge and charge densities are indeed different. As we saw from the previous problem, equal charges would guarantee a greater potential on the smaller sphere. Therefore, equal potentials will guarantee a smaller charge on the smaller sphere: $Q_A/R_A = Q_B/R_B$. However, the surface charge densities, $\sigma_A = Q_A/(4\pi R_A^2)$ and $\sigma_B = Q_B/(4\pi R_B^2)$, will not be equal. In fact, some algebra gives us the relationship $\sigma_A R_A = \sigma_B R_B$. In fact, the smaller sphere will have the greater charge density.

Again, assuming that the charges are uniformly distributed over the surface, it turns out that the electric field at the surface of the small sphere will be greater than the electric field at the surface of the large sphere. (Can you see this? I can see it two different ways.)

Consider the four charged metal spheres on the right, connected by conducting wires as shown.



Compared to the potential difference between A and B, the potential difference between C and D is

1. larger
2. the same
3. smaller
4. impossible to determine without knowing how the excess charge is distributed on the spheres

ANS: **2**—The potential difference between A and B is the same as the potential difference between C and D.

Throughout each pair of connected spheres, the potential has a constant value because they are conductors. Therefore, the potential difference between the two sets of connected spheres cannot depend upon the points chosen within.

Consider an arbitrarily shaped charged conductor. If we double the total charge on it, which of the following also doubles?

1. the electric field at point P
2. the potential at point P
3. both of the above
4. neither of the above

ANS: **3**—Both the electric field and the potential will double.

This is a very subtle and important point. The doubled charge will distribute itself spatially just as before, only the charge density everywhere will be doubled. This will produce twice as strong an electric field everywhere and thus twice the potential.

Warmup Question

Given a typical charged parallel-plate capacitor (with equal but opposite charges on the plates), what happens to the energy stored in it if the plates are pulled back to twice their original separation? If the energy changes, where does the difference come from/go to?

ANS: The capacitance decreases, while the charge remains constant. Therefore, the energy stored in the capacitor increases ($U = Q^2/2C$). In fact, the capacitance is cut in half, so the energy doubles.

The extra energy comes from the work you must do to pull the plates apart. There is an attractive force between the oppositely-charged plates, so to pull them apart you have to exert a force in the direction of the displacement (in this case, the direction of separation). The work you do on the capacitor by moving the plates apart is equal to the increase in the capacitor's energy.

Warmup Question

Capacitance is measured in farads, which is defined so that a 1 farad capacitor has ± 1 Coulomb of charge on its plates when a potential difference of 1 Volt exists between them. In practice, typical capacitors are about a centimeter square and have capacitances of a few microfarads. If you wanted a 1 farad capacitor, about how big would it have to be?

ANS: The capacitance is proportional to the area of the plates. We want to increase the capacitance by a factor of about 10^6 (one million) to get from microfarads to farads. This means that each side of the capacitor plates must be multiplied by 10^3 (one thousand). Therefore your plates should be about 10 meters square. That's a pretty big room, on the order of the size of our classroom. The parallel-plate design is not particularly good for capacitors.