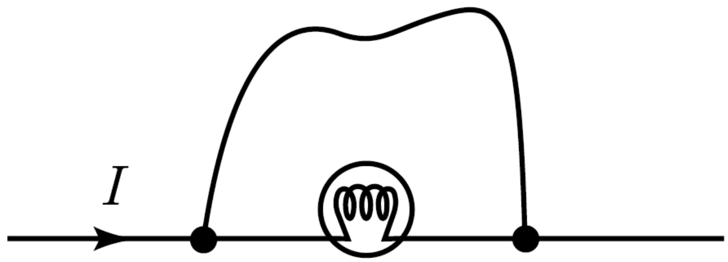
Resistors

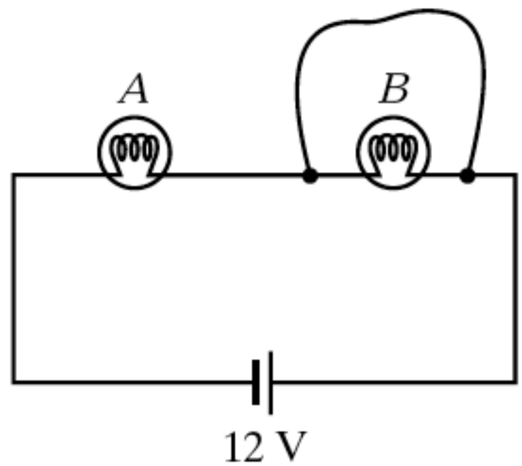
Current flows through a light bulb. Suppose a wire is connected around the bulb as shown.



When the wire is connected

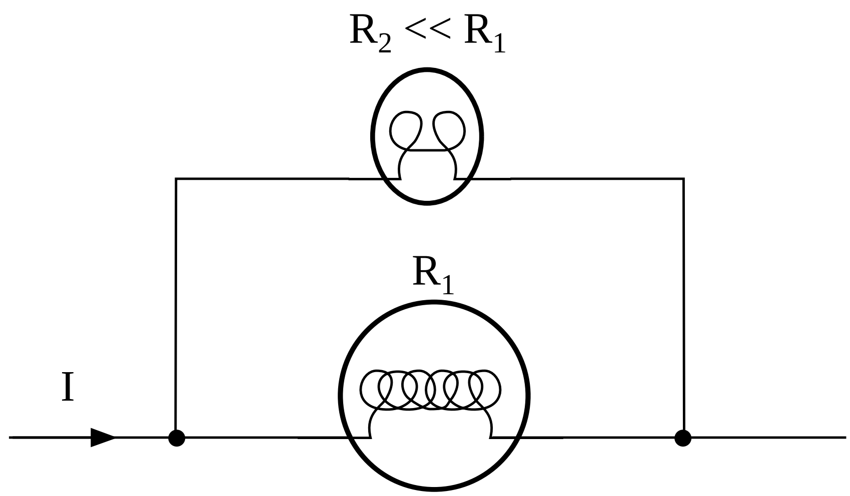
1. all the current continues to flow through the bulb
2. half the current flows through the wire, the other half continues through the bulb
3. the original amount of current flows through the wire and none through the bulb
4. some amount of current (not necessarily the same) flows through the wire and none through the bulb
5. none of the above

Two light bulbs A and B are connected in series to a constant voltage source. When a wire is connected across B as shown, bulb A



1. burns more brightly.
2. burns as brightly.
3. burns more dimly.
4. goes out.

Consider a light bulb with current flowing through it. A second light bulb with a much, much smaller resistance is added, as shown below. What happens to the current in the second case when it reaches the junction?



1. The current is split equally between the two bulbs.
2. All the current flows through the new bulb because that is the path of least resistance.
3. Most of the current flows through the new bulb, but some flows through the original bulb.
4. Most of the current flows through the original bulb, but some flows through the new bulb.
5. All the current flows through the original bulb.

ANS: **3**—Most of the current flows through the new bulb.

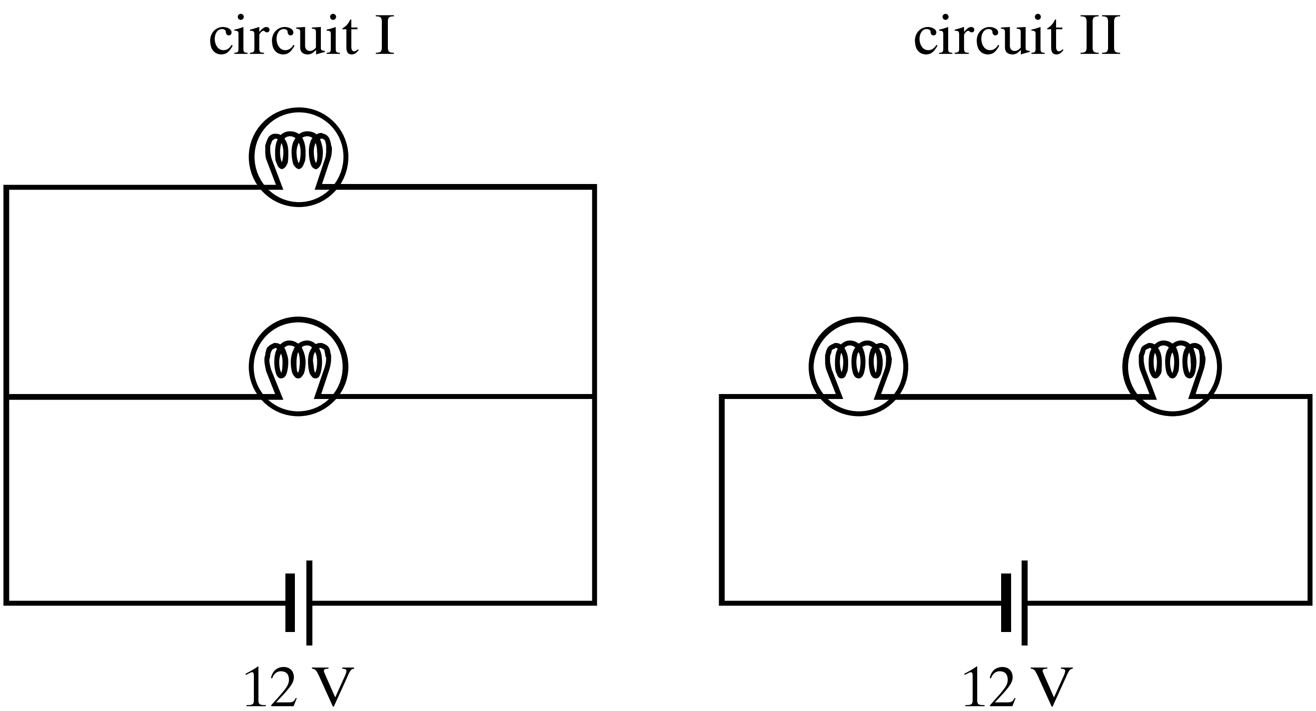
The potential difference across is the same across the two bulbs. Let's call this ΔV.

The current through the original bulb is $I_1 = \Delta V/R_1$, while the current through the new bulb is $I_2 = \Delta V/R_2$. Because $R_2 \ll R_1$, $I_2 \gg I_1$. Most of the current will go through the new, lower resistance bulb.

However, you should note that **some** current will go through the original bulb! The only way that to make no current go through $I_1 = 0$ is to make $\Delta V = 0$, which we could do by making $R_2 = 0$ (i.e. by using a wire instead of a bulb in the place of R_2).

It is easy, but incorrect, to believe the old adage that the current follows the "path of least resistance." In reality, *more* current follows the path of *less* resistance.

The four light bulbs in the figure are identical. Given that brightness is proportional to power dissipated, which circuit puts out more light?



1. I.
2. II
3. The two emit the same amount of light.

ANS: **1**—Circuit I puts out more light.

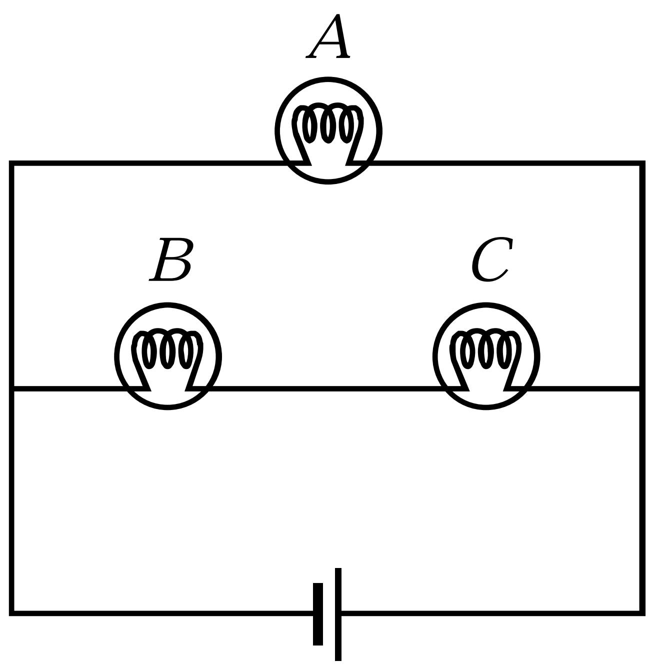
There are a number of different ways of seeing this. First, you could find the equivalent resistance of the bulbs. Suppose each bulb has resistance R. In circuit I, the equivalent resistance of the two bulbs in parallel is $R_p = R/2$. In circuit II, the equivalent resistance of the two bulbs in series is $R_s = 2R$. Therefore, the equivalent resistance in circuit I is four times less than the equivalent resistance in circuit II. That means that the battery of circuit I will deliver four times as much current as will the battery of circuit II: $I_I = 4I_{II}$. The current I_I is split between the two bulbs in circuit I, while the current I_{II} is the same for the two bulbs in circuit II. Therefore, each bulb in circuit I gets twice the current of each bulb in circuit II, and therefore will be brighter. (It will be four times brighter, because the power dissipated by a bulb is proportional to the square of the current through it.)

An easier way to look at it is to consider total power delivered by the battery as opposed to the current through each bulb. The power delivered by a battery is $P = I\Delta V$. Therefore, the battery of circuit I delivers four times as much power as that of circuit II. The total rate at which all bulbs in the circuit put out light is equal to the power delivered by the battery. This means that circuit I puts out four times as much light as does circuit II.

Finally, you can consider the power dissipated by each bulb. In circuit I, the bulbs are in parallel. Therefore, the power dissipated by each bulb in circuit I is $\Delta V^2/R$, where ΔV is the potential difference across each bulb (and the battery), and R is the resistance of the bulb, so the total power dissipated by the circuit is $P_I = 2\Delta V^2/R$. In circuit II, on the other hand, the potential difference across each bulb is $\Delta V/2$, so the power dissipated by each bulb is $(\Delta V/2)^2/R = \Delta V^2/(4R)$. Therefore the total power dissipated by the circuit is $P_{II} = \Delta V^2/(2R)$.

There are even other ways to get to the final answer. All will agree that circuit I puts out more light than circuit II.

The three light bulbs in the circuit all have the same resistance. Given that brightness is proportional to power dissipated, the brightness of bulbs B and C together, compared with the brightness of bulb A, is



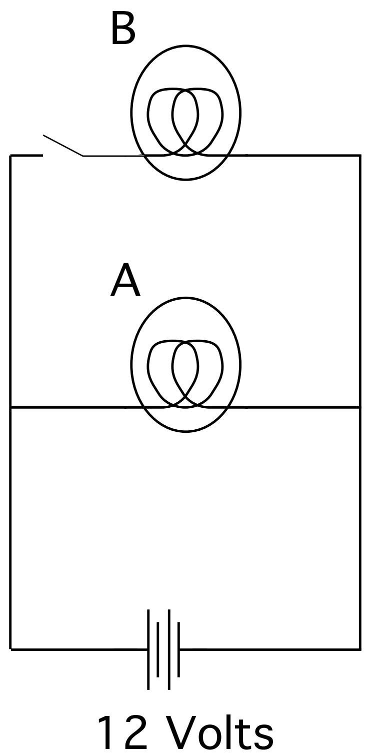
1. twice as much.
2. the same.
3. half as much.

ANS: **3**—Bulbs *B* and *C* together have half the brightness of *A*.

Because *A* is in parallel with the series combination of *B* and *C*, the potential difference across *A* will be the same as the potential difference across *B* and *C*. Therefore, the current through *B* and *C* will be half the current through *A*. The power through a resistor is proportional to the square of the current through it, so the power dissipated by *A* will be four times the power dissipated by *B*, and four times the power dissipated by *C*. Therefore, the power dissipated by *B* and *C* together will be half the power dissipated by *A*, or the total brightness of *B* and *C* together will be half the brightness of *A*.

You can also answer this question by considering the potential difference across each parallel branch. (It is the same for the *A* branch and for the *B*/*C* branch.) The power dissipated by all bulbs in a branch is the square of the potential difference across the branch divided by the equivalent resistance of the branch: $P = \Delta V^2/R$. Both branches have the same potential difference, but the *B*/*C* branch has twice the resistance of the *A* branch and therefore dissipates half as much power.

The circuit below initially consists of a light bulb **A** powered by a single 12V battery. When a second, *identical* bulb **B** is added to the circuit by closing the switch, the brightness of bulb **A**



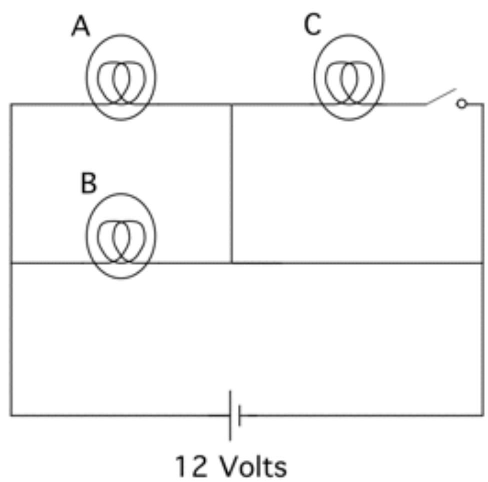
1. increases.
2. remains unchanged.
3. decreases.

ANS: **2**—The brightness of the bulb remains unchanged.

The potential difference across bulb **A** is 12V whether or not the switch is closed. Closing the switch allows additional current to pass through bulb **B**, but it does not change the current through bulb **A**.

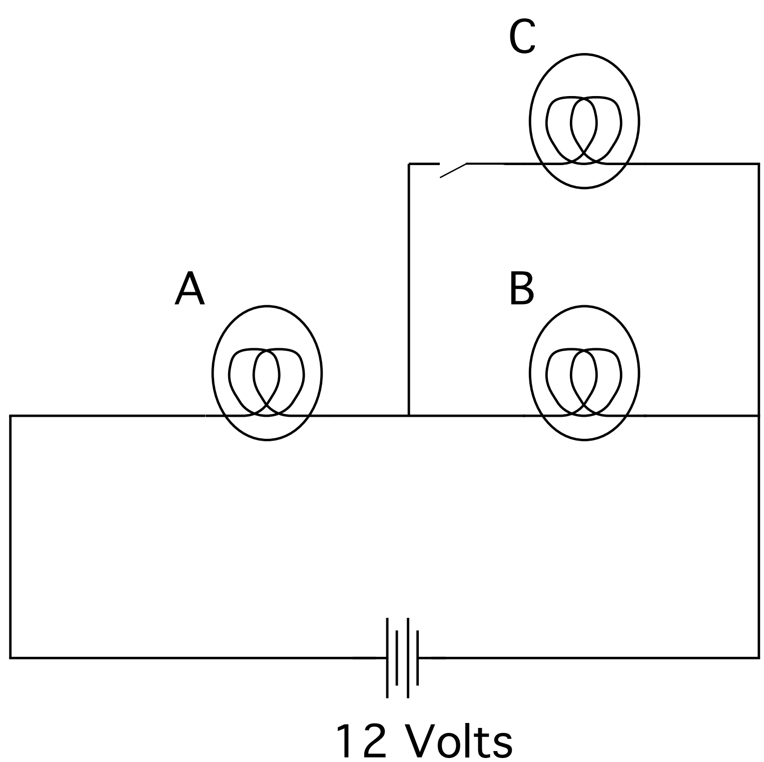
The brightness of bulb **A** is determined by the power it dissipates, which is equal to the product of the voltage across **A** and the current through it, or more appropriately in this case, $\Delta V^2/R$. This quantity does not change when the switch is closed.

The circuit below consists of two identical light bulbs A and B burning with equal brightness and a single 12 V battery. When a third identical bulb C is added by closing the switch shown, the brightness of bulb A



1. increases
2. remains unchanged
3. decreases

The circuit below consists of two identical light bulbs **A** and **B** burning with equal brightness and a single 12V battery. When a third identical bulb **C** is added, the brightness of bulb **A**



1. increases.
2. remains unchanged.
3. decreases.

ANS: **1**—The brightness of bulb **A** increases.

When the switch is closed, bulbs **B** and **C** together present less resistance to current than bulb **B** alone. Adding a reduced **B** & **C** resistance to the unchanged resistance **A** thus provides an overall lower equivalent resistance for the whole circuit, so a greater current will flow at the battery's fixed voltage. All of this current from the battery must pass through bulb **A**, so it gets brighter.

This makes the potential difference across **A** greater than before the switch was closed, which also makes the potential difference across **B** less than before the switch was closed. Therefore, bulb **B** will get dimmer when the switch is closed.

Warmup Question

A typical electric bill specifies how much electricity a household consumes in units of kilowatt-hours abbreviated kWh (i.e., 1000 watts of power times 1 hour of time). What is the corresponding proper SI unit for the quantity expressed as 1 kWh? Calculate the conversion factor between the two units. If the going rate is 10 cents/kWh, estimate the cost of raising a piano from street level to a tenth floor apartment, neglecting the cost of labor, pizza, and/or donuts. Please explain your reasoning fully and carefully.

ANS: One kilowatt of power is equal to $1000\,\mathrm{W} = 1000\,\mathrm{J/s}$.

One hour of time is equal to $3600\,\mathrm{s}$.

So $1\,\mathrm{kWh} = \left(10^3\,\mathrm{J/s}\right)\left(3.6 \times 10^3\,\mathrm{s}\right) = 3.6 \times 10^6\,\mathrm{J}$.

To find the cost of raising a piano, let's estimate the mass of a piano to be around $150\,\mathrm{kg}$ (about two people). This has a weight of around $mg = 1500\,\mathrm{N}$. A building story is about $3\,\mathrm{m}$ high, so we want to raise the piano about $30\,\mathrm{m}$. This requires a total work of $(1500\,\mathrm{N})(30\,\mathrm{m}) = 4.5 \times 10^4\,\mathrm{J}$. This equates to a little over $10^{-2}\,\mathrm{kWh}$. At a cost of $10\,\mathrm{cents/kWh}$, the electricity would cost a little over $0.1\,\mathrm{cents}$.

Warmup Question

Of the two categories, parallel and series, households in the US are wired almost exclusively in one way. Explain why that method is used and what would happen if the other method was used.

ANS: Household circuits are wired in **parallel**.

There are a couple reasons for this. Most importantly, we want to ensure that every appliance we plug into the wall is supplied with the same voltage, no matter how many other appliances are operating at the same time. We can ensure this with parallel wiring.

Secondly, if appliances are wired in series, current must pass through all of them in order to pass through any of them. Think of an older string of Christmas lights where if one bulb burns out, all lights go out. This would not be a very good design for the use of electrical devices in the home.

There is a big potential problem with parallel wiring, however. Every time you run a new device it adds to the total current delivered by the household wiring. If enough current is drawn, the wires can get hot and start a fire. The solution that household wiring uses is to have a number of small circuits, each associated with a handful of outlets. If too many items draw current at the same time, that circuit's “fuse” or “circuit breaker” will trip. This stops all current through that circuit. Putting an upper limit on the total current protects against electrical fires.