MA 207 Intro to Probability **Name:** ________________ **Date:**_____

1. Suppose you roll a fair, 6-sided die. Find the probability that your roll is a:

    a. 4 $P(4) = 1/6$

    b. 5 or 6 $P(5\ or\ 6) = 2/6 = 1/3$

    c. Even number $P(2,4,6)\ =\ 3/6$

    d. Prime number $P(2,3,5)\ =\ 3/6$

2. Suppose you roll two fair, 6-sided dice. Find the probability that the sum of your roll is a:

    a. 4 $P(4) = 3/36$

    b. 5 or 6 $P(5\ or\ 6) = (4+5)/36 = 9/36 = 1/4$

    c. Even number $P(2,4,6,8,10,12) = (1+3+5+5+3+1)/36 = 18/36 = 1/2$

    d. Prime number $P(2,3,5,7,11)\ =\ (1+2+4+6+2)/36 = 15/36$

    e. Even number and prime number $P(even\ and\ prime) = P(2) = 1/36$

    f. Even number or prime number

$$P(even\ or\ prime) = P(even) + P(prime) - P(even\ and\ prime) = \frac{1}{2} + \frac{15}{36} - \frac{1}{36} = \frac{32}{36} = \frac{8}{9}$$

3. Suppose you have a coin that is not fair because heads comes up 55% of the time when the coin is flipped.

    a. Specify the probability model for the situation.

$$P(H) = 0.55, P(T) = 0.45$$

    b. Find the probability of flipping two heads in a row.

$$P(HH) = P(H) \times P(H) = .55 \times .55 = 0.3025$$

    c. Find the probability that you will get no heads in 5 consecutive flips.

$$P(no\ Heads) = P(all\ tails) = P(T) \times P(T) \times P(T) \times P(T) \times P(T) = (.45)^5 = 0.0185$$

    d. Find the probability that you will get at least one tails in 5 consecutive flips.

Use the “It Either Happens or It Doesn’t” Theorem:

$$P(at\ least\ one\ T) = 1 - P(all\ H) = 1 - (.55)^5 = 0.9497.$$

4. Suppose you have two fair dice: one is 4-sided, and one is 12-sided.

   a. Specify the probability model for the situation.

	1	2	3	4	5	6	7	8	9	10	11	12
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	(1,7)	(1,8)	(1,9)	(1,10)	(1,11)	(1,12)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	(2,7)	(2,8)	(2,9)	(2,10)	(2,11)	(2,12)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)	(3,7)	(3,8)	(3,9)	(3,10)	(3,11)	(3,12)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)	(4,7)	(4,8)	(4,9)	(4,10)	(4,11)	(4,12)

There are $4 \times 12 = 48$ total, equally likely outcomes. The probability of each outcome is $1/48$.

   b. Find the probability of rolling a sum of 4.

   There are 3 ways to roll a 4: (3,1), (2,2), and (1,3). The probability is therefore $3/48$.

   c. Find the probability of rolling a sum of 3.

   There are 2 ways to roll a 3: (2,1) and (1,2). The probability is therefore $2/48$.

5. Consider the proposed probability model for a 4-sided die below.

$$P(1) = 0.2, P(2) = 0.3, P(3) = 0.4, P(4) = 0.3.$$

   Is this a legitimate probability model? Why or why not?

   It is not. The two requirements for a probability model are that each of the specified probabilities be between 0 and 1 (which is fine here), and that the sum of all probabilities equals one. The sum of all the probabilities here is 1.2, which is too large!

6. You have a bag of marbles that contains 4 red, 6 blue, and 10 green marbles. Besides their color, the marbles are otherwise indistinguishable.

   a. Find the probability of selecting a red marble. $P(R) = 4/20$

   b. Find the probability of selecting 3 red marbles in a row without replacement.

      Each time we select a marble without replacement, we change the probabilities for the remaining marbles. $P(RRR) = (4/20) \times (3/19) \times (2/18) = 0.0035$

   c. Find the probability of selecting 3 red marbles in a row with replacement.

With replacement the probability of drawing a red is the same each time.

$$P(RRR) = (4/20) \times (4/20) \times (4/20) = 0.008$$

d. Find the probability of selecting 1 red marble then 1 blue marble without replacement.

$$P(R \text{ and } B) = P(R) \times P(B \text{ given } R) = (4/20) \times (6/19) = 0.063$$

7. Consider a standard deck of 52 playing cards. Assume that the deck has been well-shuffled so that all cards are equally likely to be drawn.

   a. Find the probability of drawing an ace. $P(Ace) = 4/52 = 1/13$

   b. Given that an ace has already been drawn and not replaced, find the probability of drawing a king next. $P(King\ given\ Ace) = 4/51$

   c. Find the probability of drawing 3 red cards in a row (without replacement).

$$P(RRR) = (26/52) \times (25/51) \times (24/50) = 0.1176$$

8. Find the probability of flipping exactly one heads in 4 flips of a fair coin.

There are 4 ways to get one H: HTTT, THTT, TTHT, TTTH. These are disjoint outcomes, so

$$P(one\ H) = P(HTTT) + P(THTT) + P(TTHT) + P(TTTH) = 4 \times (.5)(.5)(.5)(.5) = 0.25$$

9. (Note: parts e. and g. should be pretty surprising!) Assuming a non-leap year, and assuming that all birthdates are equally likely, find the probability that:

   a. Two people in a room have different birthdays.

   There are 364 available birthdays for Person 2 that are different from Person 1's. Thus the probability is $\frac{364}{365} = 0.9973$ that the two birthdays are different.

   b. Three people in a room all have different birthdays.

   If Person 1 and Person 2 have different birthdays, then there are 363 available birthdays for Person 3 that are different from those two. Thus the probability that all three are different is $\left(\frac{364}{365}\right) \times \left(\frac{363}{365}\right) = 0.9918.$

   c. Four people in a room all have different birthdays.

   $\left(\frac{364}{365}\right) \times \left(\frac{363}{365}\right) \times \left(\frac{362}{365}\right) = 0.9836.$

d. Twenty people in a room all have different birthdays.

$$\left(\frac{364}{365}\right) \times \left(\frac{363}{365}\right) \times \left(\frac{362}{365}\right) \times \cdots \times \left(\frac{346}{365}\right) = 0.5886$$

e. In a room with twenty people, what is the probability that at least two birthdays match?

$$1 - 0.5886 = 0.4114 = 41.1\%$$

f. Forty people in a room all have different birthdays.

$$0.1088$$

g. In a room with forty people, what is the probability that at least two birthdays match?

$$1 - 0.1088 = 0.8912 = 89.1\%$$

(Hint: it may help to visualize people walking into the room one at a time and asking yourself, "What is the probability that the new person's birthday is different from everyone else's?". Excel or other technology will be helpful for parts d. and e.)