MA 207 Expected Value **Name:** ________________ **Date:**_____

1. You bet $10.00 on the flip of a fair coin. You bet that it will come up heads, and it is an even money bet (so heads means you win $10.00, tails means you lose $10.00). Find and interpret the expected value of the game. Is this a fair game?

$$EV = \$10.00 \times P(H) - \$10.00 \times P(T) = \$10.00 \times (0.5) - \$10.00 \times (0.5) = \$5.00 - \$5.00 = \$0.00$$

   If we were to play this game many, many times, on average we would break even. Because the expected value is $0, this is a fair game.

2. You roll two fair, 6-sided dice. If the sum of the next roll is a 12, you win $100, otherwise you lose $4. Find and interpret the expected value of the game. Should you play it?

$$EV = \$100 \times P(12) - \$4 \times P(not\ 12) = \$100 \times \left(\frac{1}{36}\right) - \$4 \times \left(\frac{35}{36}\right) = -\$1.11$$

   If we were to play this game many, many times, on average we would lose $1.11 per play. Because the expected value is negative, no, we should not play it.

3. A weighted coin has $P(H) = 0.53$, and $P(T) = 0.47$. If heads comes up, you win $5.00, and if tails comes up you lose $5.50. Find and interpret the expected value of the game and determine if it is fair.

$$EV = \$5 \times P(H) - \$5.50 \times P(T) = \$5 \times (0.53) - \$5.50 \times (0.43) = \$0.065$$

   If we were to play this game many, many times, on average we would win 6.5 cents per play. Because the expected value is positive, this is not a fair game. It is, however, advantageous for us.

4. Determine how much you would have to lose on tails to make the game in the previous problem a fair game.

   Let the amount we lose on tails be represented by x. We need to find the x that makes the expected value of the game 0:

$$0 = \$5 \cdot (0.53) - x(0.47)$$
$$x(0.47) = \$5 \cdot (0.53)$$
$$x = \frac{\$2.65}{0.47} \approx \$5.64$$

5. Roulette is a game of chance where players bet on where a marble will land on a spinning wheel. On an American roulette wheel, there are 38 possible slots where the marble can land: 36 spaces numbered 1-36, 18 of which are black, and 18 of which are red, and two green spaces labeled 0 and 00. A bet on "black" or a bet on "red" is an even-money bet.
    a. Find the expected value for a player who makes a $10.00 bet that the next spin will end up "black."

    $$EV = \$10 \times P(Black) - \$10 \times P(not\ Black) = \$10 \times \left(\frac{18}{38}\right) - \$10 \times \left(\frac{20}{38}\right) \approx -\$0.53.$$

    b. Interpret your result in a. from the casino's point of view.

    From the casino's point of view, they win about 53 cents for every $10 bet on black.

    c. What would a player have to win on this bet in order for it to be fair?

$$0 = x \cdot \left(\frac{18}{38}\right) - \$10\left(\frac{20}{38}\right)$$
$$\$5.263 = x \cdot .4737$$
$$x = \frac{\$5.263}{0.4737} \approx \$11.11.$$

We would have to win $11.11 in order for the game to be fair. This makes sense because in order to compensate for a lower probability of winning, we would need to win more than we bet.

6. You pay $3.00 to roll a fair, 6-sided die. Your payout is the next roll you make in $'s. So if you roll a 2 you win $2.00, if you roll a 3 you win $3.00, etc. Use expected value to determine whether you should play the game. Explain your conclusion.

This one is a little different because of the way the game is described. We have to account for the fact that we pay to play, so we aren't really "winning" money on every roll. The expected value should be our net winnings/losses, so we subtract the price to play from all outcomes:

$$EV = (\$1.00 - \$3.00)\left(\frac{1}{6}\right) + (\$2.00 - \$3.00)\left(\frac{1}{6}\right) + (\$3.00 - \$3.00)\left(\frac{1}{6}\right) + (\$4.00 - \$3.00)\left(\frac{1}{6}\right)$$
$$+ (\$5.00 - \$3.00)\left(\frac{1}{6}\right) + (\$6.00 - \$3.00)\left(\frac{1}{6}\right)$$
$$= (-\$2.00)\left(\frac{1}{6}\right) + (-\$1.00)\left(\frac{1}{6}\right) + (\$0)\left(\frac{1}{6}\right) + (\$1.00)\left(\frac{1}{6}\right) + (\$2.00)\left(\frac{1}{6}\right) + (\$3.00)\left(\frac{1}{6}\right)$$
$$= \$0.50.$$

Because the expected value is positive, yes, we should play the game.

7. You purchase a TV for \$400, and the store offers to sell you a full replacement warranty on the TV for \$50. If 3% of TV's like yours will fail and need to be replaced during the warranty period, use expected value to determine if the warranty is worth it from a purely financial point of view.

We can't just calculate the expected value when buying the warranty. We have to compare the decision to not buy the warranty to the decision to buy it. We make two expected value calculations:

Don't buy the warranty: $EV = \$0 \times P(TV\ doesn'tfail) - \$400 \times P(TV\ fails)$

$$= \$0 \cdot 0.97 - \$400 \cdot 0.03$$
$$= -\$12.00.$$

Buy the warranty: $EV = -\$50 \times P(TV\ doesn'tfail) + (\$400 - \$50) \times P(TV\ fails)$

$$= -\$50 \cdot 0.97 + \$350 \cdot 0.03$$
$$= -\$38.00.$$

Because buying the warranty has a lower expected value than not buying it, no, it is not worth it from a purely financial point of view.

8. What would the warranty in the previous problem have to cost in order for it to be a financially neutral decision for you?

We would need the two expected values (buy vs. don't buy) to be equal. Let x be the unknown cost of the warranty and solve:

$$-\$12.00 = -x(0.97) + (\$400 - x)(0.03)$$
$$-\$12.00 = -0.97x + \$12.00 - .03x$$
$$-\$24.00 = -x$$
$$\$24.00 = x$$

In order for the warranty to be a break-even proposition for us, it would have to cost \$24.00.