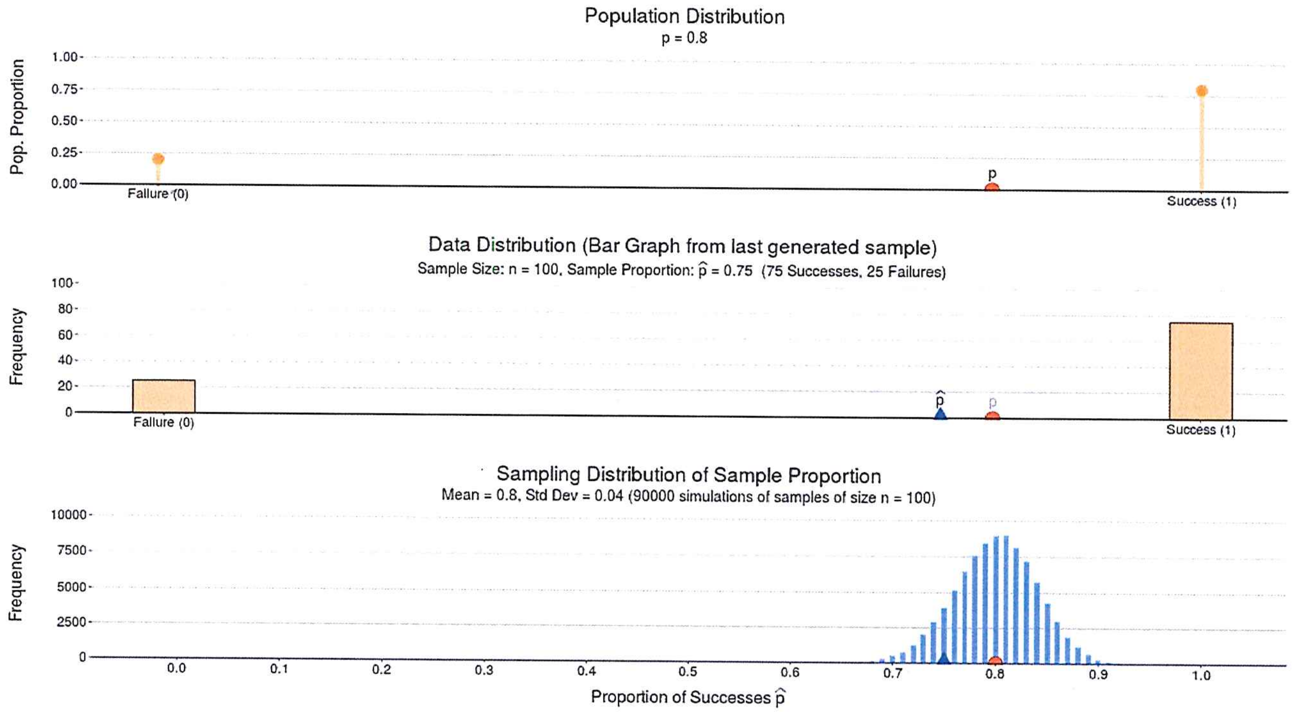
MA 207 Sampling Distribution of a Sample Proportion **Name:** ____________ ________**Date:**_____**Time:**_____

1. Suppose that 80% of cell phone charging cables work properly. Suppose we take random samples of size 100. The applet below shows the population distribution with a population proportion of $p = 0.8$, and a data distribution from just one sample that was drawn that happened to have a sample proportion of $\hat{p} = 0.75$, and the *sampling distribution* of the sample proportion which you can see is approximately normal (the sampling distribution is actually the distribution of all possible samples, but the illustration has 90,000 samples drawn).

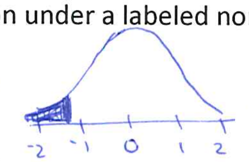


a. What is the mean of the sampling distribution of the sample proportion $\hat{p}$ when the samples are of size $n = 100$? Use the correct notation (the appropriate letter). $\mu_{\hat{p}} = p = \boxed{0.8}$

b. What is the standard deviation of the sampling distribution of the sample proportion $\hat{p}$ when the samples are of size 100? Use the correct notation (the appropriate letter). $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{100}} = \sqrt{\frac{.8(.2)}{100}} = \boxed{0.04}$

c. Suppose that you take a <u>S</u>imple <u>R</u>andom <u>S</u>ample of size 100. What is the probability that the sample proportion is less than 0.75? Shade a region under a labeled normal curve to represent your answer.

$z = \frac{0.75 - 0.8}{0.04} = -1.25$

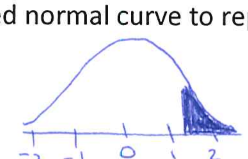


	.05
-1.2	.1056

Less than: $\boxed{0.1056}$

d. Suppose that you take a SRS of size 100. What is the probability that the sample proportion is greater than 0.85? Shade a region under a labeled normal curve to represent your answer. (Same as previous By Symmetry!)

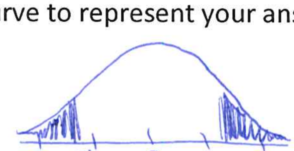
$z = \frac{0.85 - 0.8}{0.04} = 1.25$



	.05
1.2	.8944

Greater: $1 - .8944 = \boxed{0.1056}$

e. Suppose that you take a SRS of size 100. What is the probability that the sample proportion is more extreme than the sample we got (meaning the sample proportion is less than 0.75 or greater than 0.85)? Shade a region under a labeled normal curve to represent your answer. z is less than -1.25 or more than 1.25.



$0.1056 + 0.1056 = \boxed{0.2112}$

f. On the back on this paper, find $P(.7 < \hat{p} < .9)$, $P(\hat{p} < .7 \text{ or } \hat{p} > .9)$, and $P(\hat{p} > .94)$

$P(.7 < \hat{p} < .9)$ "$\hat{p}$ is greater than 0.7 and less than 0.9"

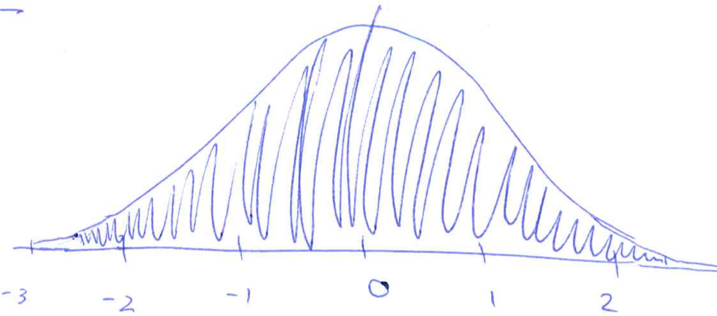
"0.7 is less than $\hat{p}$ is less than 0.9"

$$Z = \frac{.7 - .8}{.04} = -2.50$$

	.00
-2.5	.0062

$$Z = \frac{.9 - .8}{.04} = 2.50$$

	.00
2.5	.9938



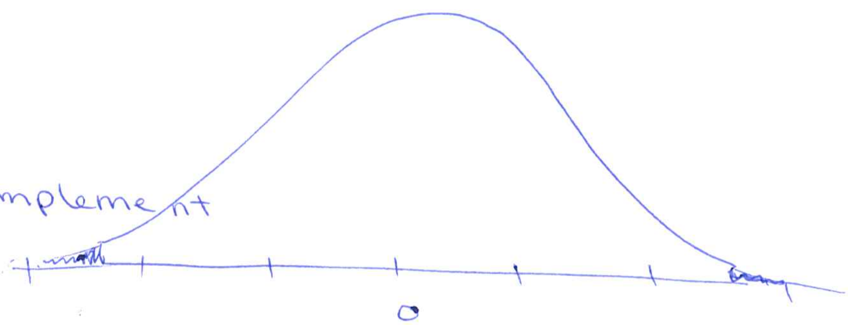
$\hat{p}$ is between .7 & .9, so the probability is $.9938 - .0062 = \boxed{.9876}$

$P(\hat{p} < .7 \text{ OR } \hat{p} > .9)$

This is the complement of above.

1 − middle

edges $= 1 - .9876 = \boxed{.0124}$



OR

$\hat{p} < .7$

	.00
-2.5	.0062

By symmetry $.0062 + .0062 = \boxed{.0124}$

$P(\hat{p} > .94)$

$$Z = \frac{.94 - .8}{.04} = 3.5$$

	.00
3.5	.999767

Greater than:

$1 - .999767 = \boxed{.000233}$

(2.33 E −4)

OR

Practically $\boxed{0}$

MA 207 Central Limit Theorem Name: ____________ ______________________ Date:______ Time:______

1. Suppose that, at a certain community college, the population distribution of the ages of the students x is approximately normal with a mean of 33 years and a standard deviation of 5 years.

$\mu = 33 \quad \sigma = 5$

a. What is the mean of the sampling distribution of sample means $\overline{x}$ when the samples are of size $n = 100$? Use the correct notation (the appropriate letter) and include units in your answer.

$\mu_{\overline{x}} = 33$ years

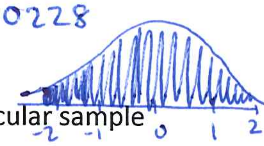
b. What is the standard deviation of the sampling distribution of sample means when the samples are of size 100? Use the correct notation (the appropriate letter) and include units in your answer.

$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{100}} = 0.5$ years

c. Suppose that you take a SRS of 100 students. What is the probability that the sample mean of this particular sample is between 32 and 34? Shade a region under a labeled normal curve to represent your answer.

$z = \frac{32-33}{0.5} = -2$ $\quad$ $z = \frac{34-33}{0.5} = 2$ $\quad$ Between: Large - Small

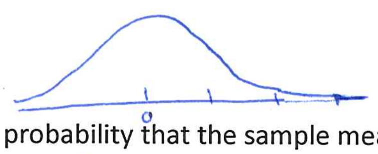
.0228 $\quad$ .9772 $\quad$ .9772 - .0228 $= \boxed{.9544}$



d. Suppose that you take a SRS of 100 students. What is the probability the sample mean of this particular sample is greater than 34.5 years? Shade a region under a labeled normal curve to represent your answer.

$z = \frac{34.5-33}{.5} = 3$ $\quad$ Greater: 1 - .9987 $= \boxed{.0013}$

.9987

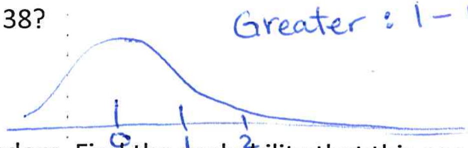


e. Suppose that you take a SRS of 100 students. Approximately what is the probability that the sample mean of this particular sample is greater than 38?

Greater: 1 - 1 = 0

$z = \frac{38-33}{.5} = 10$

Off the chart z-score

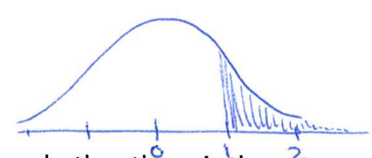


way out to here!

f. Pick one individual student at random. Find the probability that this one particular student is older than 38. Shade a region under a labeled normal curve to represent your answer. $n = 1$, so use $\sigma = 5$

$z = \frac{38-33}{5} = 1$ $\quad$ Greater/older: 1 - .8413 $= \boxed{.1587}$

.8413



g. Are your answers in part (e) and part (f) the same? Explain why or why not by describing whether there is the same variability, more variability, or less variability in the distribution of sample means compared to the distribution of the original population of students.

Not even close. The Central Limit Theorem says that Sample means have less variaton than individuals.

h. Find $P(32.5 < \overline{x} < 34)$ and $P(32.5 < x < 34)$. Which is higher and why?

$z = \frac{32.5-33}{.5} = -1$ $\quad$ .1587 $\quad$ $z = \frac{32.5-33}{5} = -0.1$ $\quad$ .4602

$z = \frac{34-33}{.5} = 2$ $\quad$ .9772 $\quad$ VS. $z = \frac{34-33}{5} = 0.2$ $\quad$ .5793

Much higher because most sample means are close to 33, Less variability

.9772 - .1587 = .8185 $\quad$ .5793 - .4602 = .1191