MA 207, §8.1 – 8.2 Confidence Intervals

MA 207 Confidence Intervals Using z-Scores Name: ____________________

1. You want to estimate the true mean height for a population of females where you know that $\sigma = 3$ inches. You take a simple random sample of 45 women and find that the average height in the sample is 65".

   a. Construct a 90% confidence interval for the true mean height of the population.

   With sample mean equal to 65, $n = 45$, and known $\sigma = 3$, we need the z^* for 90% confidence, which we find with StatCrunch to be $z^* = 1.645$. The confidence interval is therefore

   $$65 \pm 1.645\frac{3}{\sqrt{45}} = 65 \pm 0.736 = (64.264, 65.736).$$

   b. Explain what is meant by this confidence interval.

   We're 90% confident that the true mean height for females is between 64.264 inches and 65.736 inches.

   c. Determine the sample size you would need in order to be 97% sure that your sample mean is within 0.5 inches of the true mean.

   We need the margin of error to be 0.5 at the 97% confidence level. For 97% confidence, $z^* = 2.17$, so we need to solve:

   $$\begin{aligned} 2.17\frac{3}{\sqrt{n}} &= 0.5 \\ \frac{2.17 \cdot 3}{.5} &= \sqrt{n} \\ n &= 169.5 \\ n &= 170 \end{aligned}$$

2. A U.S. company manufactures about $1,000,000$ fasteners (nuts, bolts, screws, etc.) per day. With such large quantities, inevitably some of the fasteners produced are defective. A quality control test is performed on a SRS of 800 fasteners; 28 of the fasteners are defective.

   a. Construct a 95% confidence interval for the true proportion of fasteners which are defective. Explain what is meant by this confidence interval.

      With $\hat{p} = \frac{28}{800} = 0.035$, $z^* = 1.96$, and $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.0065$, the confidence interval is

      $$0.035 \pm 1.96 \cdot .0065 = 0.035 \pm .0127 = (.0223, .0477).$$

      We're 95% confident that the true proportion of defective fasteners is between 0.0223 and 0.0477.

   b. Executives decide that if the population proportion of defective fasteners is 0.05 or higher, then they will invest in new machinery to maintain product integrity. Based on this confidence interval, what do the executives conclude?

      Since the confidence interval lies entirely below 0.05, the executives can conclude with 95% confidence that the current machinery is good enough.

   c. Construct a 99% confidence interval for the true proportion of defective fasteners. Is it larger or smaller than the 95% confidence interval? Based on this confidence interval, what do the executives conclude?

      Increasing the confidence level expands the interval because we want to be more certain that the interval contains the true proportion. For 99% confidence, our new $z^* = 2.576$ and the new interval is $(0.0182, 0.0518)$ (which is larger than the previous one). The executives cannot be 99% confident that the current machinery is good enough because the replacement threshold lies in the interval. If they want to be 99% sure that no more than 0.05 of the fasteners are defective, they need to buy new machinery.

   d. How many fasteners would have to be sampled in order for the margin of error to be 0.01 with 99% confidence? (Assume you have to answer this before conducting a study.)

      We need the margin of error to be 0.01 at the 99% level, and we use $p = 0.5$ because we haven't taken a sample yet. Solve:

$$
\begin{aligned}
2.576\sqrt{\frac{.5(1-.5)}{n}} &= 0.01 \\
\frac{2.576 \cdot 0.5}{0.01} &= \sqrt{n} \\
n &= 16,589.44 \\
n &= 16,590
\end{aligned}
$$

3. An exit poll showed that 52% of 3000 respondents voted for Chris for president. Does a 95% confidence interval show that Chris will win the overall election? The confidence interval is $0.52 \pm 1.96\sqrt{\frac{.52(1-.52)}{3000}} = 0.52 \pm 0.018 = (0.502, 0.538)$. This interval is entirely above the majority needed (0.5) for Chris to win the election. We would declare Chris the winner with 95% confidence.

4. $(0.4965, 0.5435)$ is a 99% confidence interval for the population proportion of voters supporting Chris.

   a. Can Chris be declared the winner using this confidence interval? No, the interval conatins 0.5 so we can't declare Chris the winner, at least not with 99% confidence.

   b. Use the interval to determine the sample proportion and the margin of error. The sample proportion is always the middle of the interval, so $\hat{p} = 0.52$. The margin of error is the distance from the center to the endpoints of the interval, or half the length of the interval. The margin of error is therefore $0.5 \cdot (0.5435 - 0.4965) = 0.0235$.

5. Determine whether each item is true or false. Explain.

   a. For a fixed confidence level, when the sample size increases, the length of the confidence interval for a population mean decreases. True – the sample size is in the denominator of the margin of error, so as the sample size increases, the margin of error decreases.

   b. The critical value corresponding to a 90 percent confidence level is 1.96. False – 1.96 is the critical value for 95% confidence.

   c. The best point estimate for the population proportion is the sample mean. False – to estimate the population proportion use the sample proportion.

   d. The larger the level of confidence, the shorter the confidence interval. False – more confidence requires larger intervals.

e. A 90% confidence interval for a population parameter means that if a large number of confidence intervals were constructed from repeated samples, then on average, 90% of these intervals would contain the true parameter. True.

f. The point estimate, $\hat{p}$ is always at the center of the confidence interval for a population proportion. True.