9.2 One Sample Population Proportion Test

Can the difference between the sample proportion and the presumed population proportion be explained by random variability?

1. Parameter of interest: p
2. Necessary assumptions: Data obtained randomly. The sampling distribution for $\hat{p}$ is approximately normal when the number of successes (np_0) and failures ($n(1-p_0)$) are each at least 15.
3. Test statistic formula assuming the null is true: $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$

Problem 1 A research center claims that 25% of college graduates think a college degree is not worth the cost. You aren't so sure. To test this claim you ask a random sample of 200 college graduates whether they think a college degree is not worth the cost. Of those surveyed, 21% reply yes. At $\alpha = 0.10$ what do you conclude?

a. State the null hypothesis:

$H_0 : p = 0.25$

b. State the alternative hypothesis:

$H_A : p \neq 0.25$

c. Explicitly check any assumptions needed to perform the significance test:

We need at least 15 expected successes and 15 expected failures in the sample: $np_0 = 200 \cdot 0.25 = 50 \geq 15$, and $n(1-p_0) = 200 \cdot 0.75 = 150 \geq 15$.

d. Determine the test statistic: $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \dfrac{0.21 - 0.25}{\sqrt{\dfrac{.25(1-.25)}{200}}} = \frac{-0.04}{.0306} = -1.31$

e. Is this a left tailed, right tailed, or two tailed test? Two-tailed since a particular direction isn't specified – just that we're skeptical of the claim.

f. Draw a labeled curve and shade the appropriate region:

g. Determine the P-Value: The P-value is for a two-tailed test, so we find $2 \times P(z \leq -1.31) = 2 \times 0.095 = 0.19$.

h. Explain the meaning of the P-Value (You are not using α yet): If the null hypotheis were true, we would get a sample proportion at least as extreme as 0.21 19% of the time just by chance.

i. Compare P with α: $P = 0.19 > 0.10 = \alpha$.

j. Describe the conclusions: Since the P-value is larger than α, we do not have sufficient evidence to reject the null so we fail to reject the null.