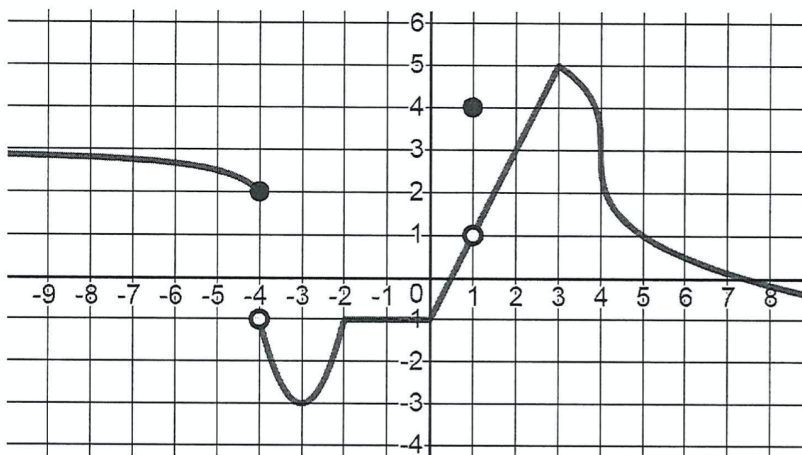


Sample Exam 1

1. Given the graph of $f(x)$, give a numerical value or state that the value does not exist.

[37]



(2 points each):

$f(-4) =$

$f(1) =$

$\lim_{x \rightarrow -4^-} f(x) =$

$\lim_{x \rightarrow 1^-} f(x) =$

$\lim_{x \rightarrow -4^+} f(x) =$

$\lim_{x \rightarrow 1^+} f(x) =$

$\lim_{x \rightarrow -4} f(x) =$

$\lim_{x \rightarrow 1} f(x) =$

$\lim_{x \rightarrow -\infty} f(x) =$

$f'(-3) =$

$\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} =$

$f'(1) =$

$\lim_{x \rightarrow -2^+} f'(x) =$

$f'(2) =$

(3 points each): For the graph of $f(x)$ above, answer the following questions.

Is $f(x)$ continuous at $x = 4$? Briefly explain.

Is $f(x)$ differentiable at $x = 4$? Briefly explain.

Will $f'(6)$ be positive, negative, zero, or not exist? Briefly explain.

2. Evaluate the following limits. Give a numerical answer, if it exists. If the limit is infinite, write ∞ or $-\infty$ as appropriate. If the limit does not exist, write DNE.

Show your work and justify your answers using algebra! Do NOT use a table of values or a graph to justify your answers.

(a) $\lim_{x \rightarrow 3} \frac{2x^2 - 6x}{x + 3}$ [3]

(b) $\lim_{x \rightarrow 2^+} \frac{x^2 + 1}{2 - x}$ [3]

(c) $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x + 2}$ [3]

(d) $\lim_{x \rightarrow 1} \ln(x) - \sqrt{x}$ [3]

3. (a) Finish the definition of the derivative started below:

[4]

Given a function $f(x)$, we define the **derivative** of $f(x)$ to be the limit, if it exists,

$$f'(x) =$$

- (b) Calculate the derivative of $f(x) = 3x^2 - x$ using the definition.

[8]

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4. Given that $f(x) = x^2(2x - 1)$ and $f'(x) = x(6x - 2)$, find the equation of the tangent line to the graph of f at $x = 2$. [4]
5. Let $T = f(A)$ be the temperature T of the atmosphere in degrees Celsius at an altitude of A kilometers above the surface of the earth.
- (a) What does $T(60) = -33$ mean in the context of the above scenario? [4]
- (b) What does $T'(60) = -\frac{1}{5}$ mean? [4]
- (c) What does $T^{-1}(10) = 3$ mean? [4]
- (d) What does $(T^{-1})'(10) = 4$ mean in English? [4]
- (e) At 90 km above the earth's surface, the temperature will rise by 2 degrees Celsius for every additional kilometer moved away from the earth's surface. How would you write this fact in mathematical notation? [4]

6. Let $f(x) = \begin{cases} 6 - x & x < 1, \\ 3 & x = 1 \\ 3 + 2x & x > 1. \end{cases}$

(a) Use limits to determine if $f(x)$ is continuous at $x = 1$.

[4]

(b) Is $f(x)$ differentiable at $x = 1$? Explain why or why not.

[4]

7. Sketch the graph of a SINGLE function $f(x)$ which satisfies ALL of the following conditions:

[7]

1. $f(2) = 3$,
2. $\lim_{x \rightarrow 2} f(x) = -1$,
3. $f(-3) = 1$,
4. $\lim_{x \rightarrow -3^+} f(x) = 4$,
5. $f'(x) > 0$ on $-\infty < x < -3$ and $2 < x < \infty$
6. $f'(x) < 0$ on $-3 < x < 2$.

