Sample Exam III

Directions: You have one class period to complete this exam. It is due at the end of class in your instructor's office. You may use your calculators, but you may not use any books or notes. Please store your cell-phone and do not use Google. Also, you must work alone.

I, _____, am fully aware of and have abided by the BSC Honor Code in completing this exam.

Please work in the space provided and ***SHOW ALL OF YOUR WORK!***

The following geometric formulas may be useful:

The volume of a cone is $V = \frac{1}{3}\pi r^2 h$

The volume of a sphere is $V = \frac{4}{3}\pi r^3$ and has a surface area of $A = 4\pi r^2$.

The volume of a cylinder is $V = \pi r^2 h$ and has a surface area is $A = 2\pi r^2 + 2\pi r h$.

The area of a circle is $A = \pi r^2$.

The Pythagorean Theorem states that the length of the hypotenuse squared is the sum of the square of the lengths of the other two sides. That is $a^2 + b^2 = c^2$ where a and b are the lengths of the sides of a right triangle while c is the length of the hypotenuse

1. (5 pts each) Calculate the derivative of the following functions. DO NOT SIMPLIFY.

   (a) $f(x) = 2\tan(x) + \arctan(5x)$

   (b) $f(x) = \arcsin(x^2)$

   (c) $f(x) = x^3 \ln(x^2 + 1)$

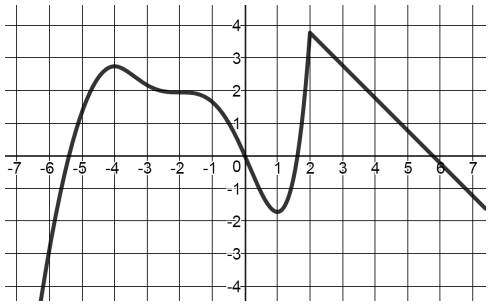
   (d) $f(x) = \dfrac{x^2 - \cos(x)}{e^x + \sin x}$

   (e) $f(x) = \ln\left(\sin(\sqrt{x^3 + 1})\right)$

2. (4 pts) In your international relations class you hear the following statement: "The world's proven oil reserves continues to rise but at a slower rate than in the last decade."

   What does this statement say about the signs of the first and second derivative of the world's proven oil reserves $R(t)$ as a function of time, t?

3. The graph of $f(x)$ is given below. Answer the following about $f(x)$. Assume that $f(x)$ is defined for all x and that the graph does not change concavity outside the window shown.



(a) (5 pts) List the x-coordinates of all critical points of $f(x)$.

(b) (4 pts) List the x-coordinates of any local **MAXIMA** of $f(x)$.

(c) (4 pts) Does $f(x)$ have a global **MAXIMUM**? If so, give the x-coordinate of the global maximum and explain why it is the maximum. If not, explain why no global maximum exists.

(d) (4 pts) Does $f(x)$ have a global **MINIMUM**? If so, give the x-coordinate of the global minimum and explain why it is the minimum. If not, explain why no global minimum exists.

(e) (4 pts) Suppose $f(x)$ is restricted to the domain $-6 \leq x \leq 0$. List all critical points of $f(x)$ on this restricted domain.

(f) (4 pts) Suppose $f(x)$ is restricted to the domain $-6 \leq x \leq 0$. In this case, does $f(x)$ have a global **MINIMUM** on this restricted domain? If so, give the x-coordinate of the global minimum and explain why it is the minimum. If not, explain why no global minimum exists.

4. (4 pts) Sketch a graph of $f(x)$ which satisfies all of the following conditions:

   (a) $f(0) = 0$,

   (b) $f'(x) > 0$ on $-2 < x < 3$ and $6 < x < \infty$

   (c) $f'(x) < 0$ on $-\infty < x < -2$, $3 < x < 6$,

   (d) $f''(x) > 0$ on $-\infty < x < 0$ and $4 < x < \infty$,

   (e) $f''(x) < 0$ on $0 < x < 4$.

5. Suppose $f(x)$ is invertible and the line $y = 3x + 2$ is tangent to $f(x)$ at $x = 2$.

(a) (3 pts) Find the slope of the tangent line to $f^{-1}(x)$ at $x = 8$.

SKIP #5a

(b) (3 pts) Find the equation of the tangent line to $f^{-1}(x)$ at $x = 8$.

SKIP #5b

6. Consider $f(x) = x^5 - 15x^3$.

(a) (4 pts) Find all critical points of $f(x)$.

(b) (4 pts) Classify each critical point obtained in part (a) as a local maximum, local minimum or neither.

(c) (4 pts) Find the global **MAXIMUM** of $f(x)$ on the restricted domain $-4 \leq x \leq 4$.

(d) (4 pts) Find all inflection points of $f(x)$.

7. A child has an 18 inch piece of yarn that she is using in the corner of a room to outline a rectangular 'pigpen' for her toy piggies. She does not use yarn against the walls. What is the area of the largest such pen she can make?

   (a) (3 pts) Draw a picture and label variables.

   (b) (5 pts) Write the quantity to be optimized as a function of one variable and find the domain of this function.

   (c) (4 pts) Use calculus to optimize the function from part (b). Be sure to test any critical points.

   (d) (3 pts) Interpret your answer in the context of the above scenario using units.

8. You manage a packaging company and have signed a contract to supply a particular type of box to a customer. The deal stipulates a minimum order of 1000 boxes per month, but the customer may order up to 2000 boxes per month. The price per box depends on the number ordered with the price starting at \$1.50 and reducing by 1/10 of a penny (or \$0.001) for every box over 1000 (on the whole order).

   (a) (1 pt) How much revenue do you make if the customer orders exactly 1000 boxes?

   (b) (2 pts) How much revenue do you make if the customer orders exactly 1300 boxes?

   (c) (5 pts) Let x be the number of boxes ordered over 1000. Give a formula for the revenue as a function of x and find the domain of this function.
   YOU DO NOT NEED TO ACTUALLY OPTIMIZE THIS FUNCTION. JUST SET UP THE EQUATION.

9. (6 pts) While studying for your calculus test, a classmate asks, "I have only four critical points, $x = 1, 2, 3$ & 4. We are told that $x = 1$ and $x = 3$ are local maxima. That makes the other two minima right?" What is your answer? (Hint: a number line may help).