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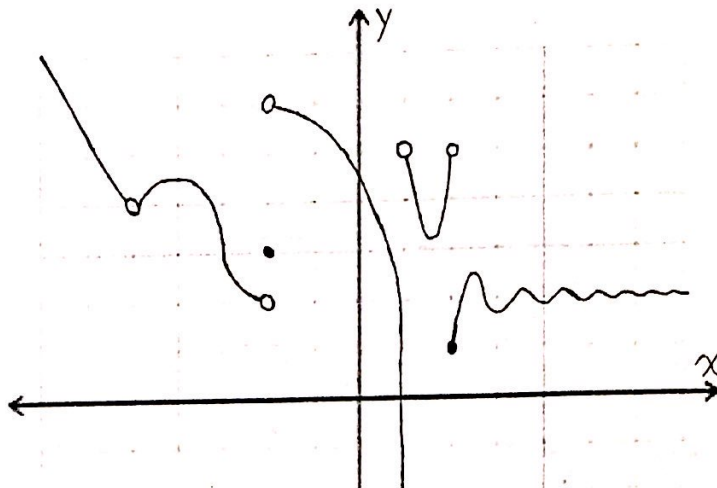
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Name:

Math 231 – Calculus I Worksheet #1: Continuity

Use the following graph for #1-2



- 1) List the x-value of all coordinates where the function graphed above is NOT continuous.

$$x = -5, x = -2, x = 1, x = 2$$

- 2) Pick one x-value from your answer in #1 and explain why the graph is not continuous at that point using the definition of continuity.

See extra page

- 3) Sketch the graph of ONE FUNCTION that meets all of the following conditions:

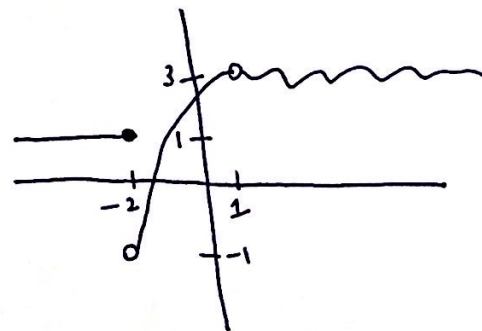
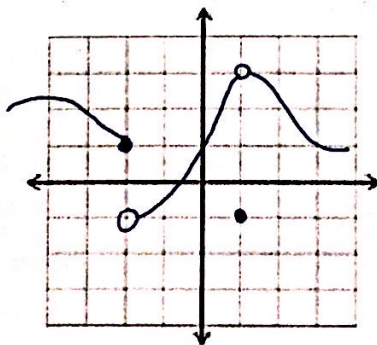
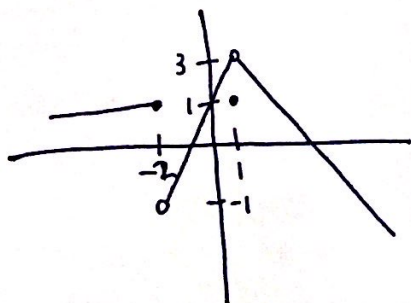
$$f(-2) = 1,$$

$$\lim_{x \rightarrow -2^-} f(x) = 1,$$

$$\lim_{x \rightarrow -2^+} f(x) = -1$$

$$\lim_{x \rightarrow 1} f(x) = 3,$$

f is NOT continuous at $x = 1$



There are many others, as well?

4) For the following function:

- a) Compute $f(c)$ for the indicated value $x = c$. Write undefined if this value does not exist.
- b) Compute the left-hand limit, the right-hand limit, and the two-sided limit at the indicated x -value. If the limit does not exist, write DNE.
- c) Decide whether the function is continuous at the indicated x -value. Justify your answer using the definition of continuity and your calculations from parts a) and b).

$$f(x) = \begin{cases} \sqrt{x^3 - 1}, & x \leq 1 \\ e^{1-x}, & x > 1 \end{cases} \quad \text{at } x = 1$$

$$(a) f(1) = \sqrt{1^3 - 1} = \sqrt{0} = 0$$

$$(b) \lim_{x \rightarrow 1^-} (\sqrt{x^3 - 1}) = \sqrt{(1)^3 - 1} = 0$$

$$\lim_{x \rightarrow 1^+} (e^{1-x}) = e^{1-1} = e^0 = 1$$

$$\lim_{x \rightarrow 1} f(x) = \text{DNE}$$

c) f is not continuous at $x = 1$

because $\lim_{x \rightarrow 1} f(x) \text{ DNE}$

$$\left[\text{so } \lim_{x \rightarrow 1} f(x) \neq f(1) \right]$$

2 $x = -5$

$$\lim_{x \rightarrow -5} f(x) = 4 \quad \text{BUT } f(-5) \text{ is undefined} \quad \lim_{x \rightarrow -5} f(x) \neq f(-5)$$

Removable discontinuity

$x = -2$

$$\lim_{x \rightarrow -2^-} f(x) = 2$$

$$\lim_{x \rightarrow -2^+} f(x) = 6$$

So this means

$$\lim_{x \rightarrow -2} f(x) \text{ DNE}$$

jump discontinuity

$$x \rightarrow -2$$

$$\text{and } f(-2) = 3$$

$$\text{So } \lim_{x \rightarrow -2} f(x) \neq f(-2)$$

$x = 1$

$$\lim_{x \rightarrow 1^-} f(x) = -\infty$$

infinite discontinuity

$$f(1) = \text{undefined}$$

$$\lim_{x \rightarrow 1^+} f(x) = 5$$

$$\lim_{x \rightarrow 1} f(x) \text{ DNE}$$

$$x \rightarrow 1$$

$x = 2$

$$\lim_{x \rightarrow 2^-} f(x) = 5$$

$$f(2) = 1$$

$$\lim_{x \rightarrow 2^+} f(x) = 1$$

$$\lim_{x \rightarrow 2} f(x) \text{ DNE}$$

jump discontinuity