**Name: \_\_\_\_\_­­­­­­­­­­­­­­­­­­­­­­­­­­\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**MA 209 Lab 6: Intro to Probability**

1. Suppose you roll a fair, 6-sided die. Find the probability that your roll is a:
   1. 4
   2. 5 or 6
   3. Even number
   4. Prime number
2. Suppose you roll two fair 6-sided dice. Write down the sample space of all possible outcomes.

Table

Description automatically generated

1. Suppose you roll two, fair 6-sided dice. Find the probability that the *sum* of your roll is a:
   1. 4
   2. 5 or 6 (This is a union of two mutually exclusive events, so we use the sum rule.)
   3. Even number (The event “even number” can be written as the union of 6 mutually exclusive events, so we use the sum rule.)
   4. Prime number (The event “prime number” can be written as the union of 6 mutually exclusive events, so we use the sum rule.)
   5. Even number and prime number (Here we’ve noticed that the event “even and prime” only includes one outcome – rolling a 2. We could also do this using conditional probability:
   6. Even number or prime number
2. Suppose you have a coin that is not fair because heads comes up 55% of the time when the coin is flipped. Note that coin flips are independent, so we can use the simple multiplication rule throughout.
   1. Specify the probability model for the situation.
   2. Find the probability of flipping two heads in a row.
   3. Find the probability that you will get no heads in 5 consecutive flips.
   4. Find the probability that you will get at least one tails in 5 consecutive flips.

Use the law of total probability with the complement of the event “at least one tail” being “no tails” or “all heads”:

1. Suppose you have two fair dice: one is 4-sided, and one is 12-sided.
   1. Specify the probability model for the situation.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) | (1,7) | (1,8) | (1,9) | (1,10) | (1,11) | (1,12) |
| 2 | (2,1) | (2,2) | (2,3) | (2,4) | (2,5) | (2,6) | (2,7) | (2,8) | (2,9) | (2,10) | (2,11) | (2,12) |
| 3 | (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3,6) | (3,7) | (3,8) | (3,9) | (3,10) | (3,11) | (3,12) |
| 4 | (4,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) | (4,7) | (4,8) | (4,9) | (4,10) | (4,11) | (4,12) |

There are total, equally likely outcomes. The probability of each outcome is .

* 1. Find the probability of rolling a sum of 4.

There are 3 ways to roll a 4: (3,1), (2,2), and (1,3). The probability is therefore .

* 1. Find the probability of rolling a sum of 3.

There are 2 ways to roll a 3: (2,1) and (1,2). The probability is therefore .

1. You have a bag of marbles that contains 4 red, 6 blue, and 10 green marbles. Besides their color, the marbles are otherwise indistinguishable.
   1. Find the probability of selecting a red marble.
   2. Find the probability of selecting 3 red marbles in a row without replacement.

Each time we select a marble without replacement, we change the probabilities for the remaining marbles.

* 1. Find the probability of selecting 3 red marbles in a row with replacement.

With replacement the probability of drawing a red is the same each time.

1. Consider a standard deck of 52 playing cards. Assume that the deck has been well-shuffled so that all cards are equally likely to be drawn.
   1. Find the probability of drawing an ace.
   2. Given that an ace has already been drawn and not replaced, find the probability of drawing a king next.
   3. Find the probability of drawing 3 red cards in a row (without replacement).
2. Find the probability of flipping exactly one heads in 4 flips of a fair coin.

. Or, we can write

1. Construct a Venn diagram for the following information: 55% of people enjoy romantic comedies, 57% of people enjoy action movies, and 15% of people enjoy neither.

.15

.30

.27

.28

1. Based on the previous problem, what percent of people enjoy romantic comedies and action movies? 0.27
2. Assuming a non-leap year, and assuming that all birthdates are equally likely, find the probability that:
   1. Two people in a room have different birthdays.

There are 364 available birthdays for Person 2 that are different from Person 1’s. Thus the probability is that the two birthdays are different.

* 1. Three people in a room all have different birthdays.

If Person 1 and Person 2 have different birthdays, then there are 363 available birthdays for Person 3 that are different from those two. Thus the probability that all three are different is .

* 1. Four people in a room all have different birthdays.

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* 1. Twenty people in a room all have different birthdays.
  2. In a room with twenty people, what is the probability that at least two birthdays match?
  3. Forty people in a room all have different birthdays.
  4. In a room with forty people, what is the probability that at least two birthdays match?

(Hint: it may help to visualize people walking into the room one at a time and asking yourself, “What is the probability that the new person’s birthday is different from everyone else’s?”. Excel or other technology will be helpful for parts d. and e.)