Worksheet 3 Key

If $y = f(g(x))$,
then $y' = f'(g(x)) \cdot g'(x)$

1. a) $y = e^{(2x^3+x)}$

$$\boxed{y' = e^{(2x^3+x)} \cdot (6x^2+1)}$$

b) $y = \dfrac{x^3-4}{2x^2+4e^x}$

$$y' = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{(g(x))^2}$$

$$\boxed{y' = \frac{(2x^2+4e^x)(3x^2) - (x^3-4)(4x+4e^x)}{(2x^2+4e^x)^2}}$$

c) $y = \dfrac{1}{3-4x}$

rewrite as $y = (3-4x)^{-1}$ → chain rule

$$y' = -(3-4x)^{-2}(-4) = \boxed{\frac{4}{(3-4x)^2}}$$

d) $y = 2x\left(\frac{1}{x} + \sqrt{x}\right)$ → product rule

$f'(x)g(x) + g'(x)f(x)$

$$y' = \boxed{2x\left(-\frac{1}{x^2} + \frac{1}{2}x^{-1/2}\right) + \left(\frac{1}{x}+\sqrt{x}\right)2}$$ Don't have to simplify

optional simplification $= -\dfrac{2x}{x^2} + \dfrac{2x}{2\sqrt{x}} + \dfrac{2}{x} + 2\sqrt{x}$

$= -\dfrac{2}{x} + \sqrt{x} + \dfrac{2}{x} + 2\sqrt{x}$

$$\boxed{3\sqrt{x}}$$

OK $y = 2x\left(\frac{1}{x}+\sqrt{x}\right)$ distribute 1st

$y = 2 + 2x^{3/2}$

$y' = 0 + 3x^{1/2}$

$= 3\sqrt{x}$

e. $y = \dfrac{(3x^2+x)^3}{4}$

$y = \frac{1}{4} \cdot (3x^2+x)^3$

$y' = \frac{1}{4} \cdot \frac{d}{dx} (3x^2+x)^3$

$\frac{1}{4} \cdot 3(3x^2+x)^2 \cdot (6x+1)$

$= \dfrac{3(3x^2+x)^2(6x+1)}{4}$ optional simplification

f. $y = 5\sqrt{x^2+8x+25}$

$y' = 5 \cdot \frac{1}{2}(x^2+8x+25)^{-1/2} \cdot (2x+8)$

$= \dfrac{5(2x+8)}{2\sqrt{x^2+8x+25}}$ optional simplification

$= \dfrac{5(x+4)}{\sqrt{x^2+8x+25}}$ optional simplification

g. $y = ((4x^5-1)(2x+1))^{1/3}$

$y = ((8x^6+4x^5-2x-1))^{1/3}$

$y' = \frac{1}{3}(8x^6+4x^5-2x-1)^{-2/3} \cdot (48x^5+20x^4-2)$

$\boxed{\dfrac{48x^5+20x^4-2}{3(8x^6+4x^5-2x-1)^{2/3}}}$ optional simplification

h. $y = \dfrac{(3x^2-4x+10)\sqrt{5-x}}{(x^2+1)}$

$y' = \dfrac{(x^2+1)\left[(3x^2-4x+10)\left(\frac{1}{2}\sqrt{5-x}\right)^{-1/2} \cdot -1 + \sqrt{5-x} \cdot (6x-4)\right] - (3x^2-4x+10)\sqrt{5-x} \cdot 2x}{(x^2+1)^2}$

2 a) $h(x) = \frac{f(x)}{g(x)}$

$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

$h'(0) = \frac{2 \cdot {}^{-}2 - 3 \cdot 1}{2^2} = \frac{-4-3}{4} \boxed{\frac{-7}{4}}$

b) $h(x) = f(g(x))$

$= f'(g(x)) \cdot g'(x)$

$h'(0) = f'(g(0)) \cdot g'(0)$

$= f'(2) \cdot 1$

$= 4 \cdot 1 = \boxed{4}$

c) $h(x) = 9f(x) - 2g(x)$

$h'(2) = 9f'(2) - 2g'(2)$

$9 \cdot 4 - 2 \cdot 2$

$36 - 4 = \boxed{32}$

d) $h(x) = (x^2+9)g(x)$

$h'(x) = (x^2+9)g'(x) + g(x) \cdot 2x$

$h'(2) = (2^2+9) \cdot 2 + 6 \cdot 2(2)$

$26 + 24 = \boxed{50}$

#3) slope : $f'(x) = \dfrac{\sqrt{2x-3}(1) - (x+1)(1/2)(2x-3)^{-1/2}(2)}{(\sqrt{2x-3})^2}$

$f'(2) = \dfrac{\sqrt{2(2)-3} - (2+1)(\cancel{1/2})(2(2)-3)^{-1/2}(\cancel{2})}{(\sqrt{2(2)-3})^2}$

$= \dfrac{\sqrt{1} - (3)(1)^{-1/2}}{1}$

$= \dfrac{1-3(1)}{1} = -2$

point : $(2,3)$

equation : $y = -2(x-2)+3$.

#4) $P(n) = 0.25(0.5n^2+5n+25)^{1/2}$

$P'(n) = 0.25(1/2)(0.5n^2+5n+25)^{-1/2}(0.5(2n)+5)$

$P'(12) = 0.25(1/2)(0.5\cdot 12^2+5(12)+25)^{-1/2}(0.5(2\cdot 12)+5)$

n is in thousands

≈ 0.17 $\dfrac{\text{parts per million}}{\text{thousand residents}}$

since positive, we know level of pollutant is increasing when the population is 12,000.