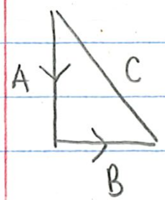
Remember: your sentence at the end (to answer the question) must include units so if it's helpful to you, include them along the way!

Worksheet 5 Key

1



$A = 12$
$B = 9$
$C = 15$

use pythagorean $A^2 + (9)^2 = 15^2$
$A = 12$

$\frac{dA}{dt} = ?$ $\quad \frac{dB}{dt} = 4 \text{ ft/sec}$ $\quad \frac{dc}{dt} = 0$

using $A^2 + B^2 = c^2$

$$\frac{d}{dt}(A^2 + B^2) = \frac{d}{dt}(c^2)$$

$$2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2c\frac{dc}{dt}$$

$$2(12)\frac{dA}{dt} + 2(9)(4) = 2(15)(0)$$

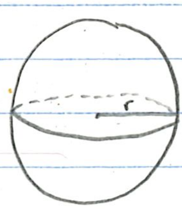
$$24\,dA/dt + 72 = 0$$

$$24\,dA/dt = -72$$

$$\frac{dA}{dt} = \frac{-72}{24} = -3 \text{ ft/sec}$$

When the base of the ladder is 9 feet from the wall, the ladder is sliding down the wall at a rate of 3 ft/sec

2



$r = \frac{D}{2}$

D = diameter
V = volume

$$V = \frac{4}{3}\pi r^3$$
$$V = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$
$$V = 4/3\,\pi\left(\frac{D^3}{8}\right)$$
$$V = \frac{4}{3}\pi \frac{1}{8} \cdot D^3$$
$$V = \frac{4}{24}\pi D^3$$
$$V = \frac{\pi}{6} D^3$$

$D = 10$ $\quad \frac{dD}{dt} = 4$ $\quad \frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{\pi}{6}D^3\right)$

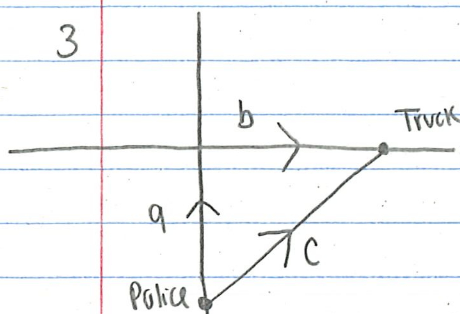
$V =$ $\quad \frac{dV}{dt} = ?$ $\quad \frac{dV}{dt} = \frac{\pi}{6}(3D^2)\frac{dD}{dt} = \frac{\pi}{2}D^2\frac{dD}{dt}$

make negative (decreasing lung capacity)

$\frac{dV}{dt} = \frac{\pi}{2}(10^2)(4) = 200\pi$ $\quad$ L = lung capacity $\quad \frac{dL}{dt} = -200\pi \text{ in}^3\text{/sec}$

When the diameter of the balloon is 10 inches, her lung capacity is decreasing at a rate of 200π in³/sec

3



$a = 30 \quad \frac{da}{dt} = -22$

$b = 40 \quad \frac{db}{dt} =$

$c = 50 \quad \frac{dc}{dt} = 9$

Find C $30^2 + 40^2 = C^2 \qquad c = 50$

$$\frac{d}{dt}(a^2+b^2) = \frac{d}{dt}(c^2)$$

$$= 2a\,{da}/dt + 2b\,{db}/dt = 2c\,{dc}/dt$$

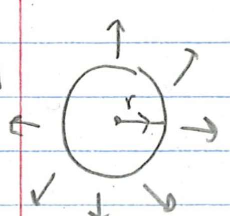
$$2(30)(-22) + 2(40)\,{db}/dt = 2(30)(9)$$

$$\frac{db}{dt} = 27.75 \text{ m/s}$$

The truck is speeding → going 27.75 m/s in a 20 m/s

He will probably get a ticket

4



$r = 10$

$\frac{dr}{dt} = 3$

$A = \quad \frac{dA}{dt} = ?$

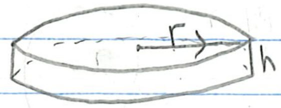
$$\frac{d}{dt}(A = \pi r^2)$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(10)(3)$$

$\frac{dA}{dt} = 60\pi$ ft^2/min The area of the oil spill is increasing at a rate of 60π ft²/min when the radius is 10 ft.

5



$r = 150 \quad \frac{dr}{dt} = .1 \text{ m/min}$

$V \quad \frac{dv}{dt} = 0$ (fixed volume)

$h = .02 \quad \frac{dh}{dt} = ?$

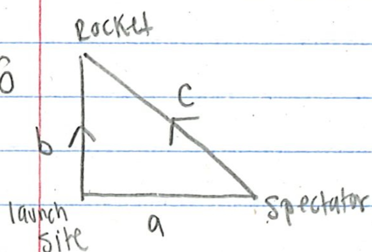
$\frac{d}{dt}(V = \pi r^2 h) \rightarrow \frac{dv}{dt} = \pi r^2 \frac{dh}{dt} + 2\pi r \frac{dr}{dt} h$ (product rule)

$0 = \pi(150^2)\, dh/dt + 2\pi(150)(.1)(.02)$

$\frac{dh}{dt} = -0.000027$ m/min

The thickness of the oil is decreasing at a rate of 0.000027 m/min

6



$a = 12000 \quad \frac{da}{dt} = 0$

$b = 5000 \quad \frac{db}{dt} = ?$

$c = 13000 \quad \frac{dc}{dt} = 480$

To find b

$12000^2 + b^2 = 13000^2$

$b = 5000$

$\frac{d}{dt}(a^2 + b^2 = c^2)$

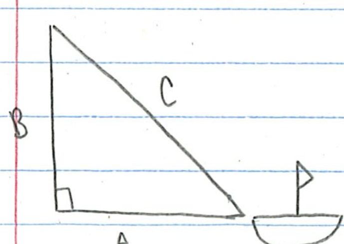
$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

$2(12000)(0) + 2(5000)\, db/dt = 2(13000)(480)$

$\frac{db}{dt} = 1248$ ft/sec

When the rocket is 13,000 ft from the spectator and increasing at a rate of 480 ft/sec, the rocket's speed is 1248 ft/sec

7



Find A $(25)^2 = 7^2 + A^2 \quad A = 24$

$A = 24 \quad \frac{dA}{dt} = ?$

$B = 7 \quad \frac{db}{dt} = 0$

$C = 25 \quad \frac{dc}{dt} = -0.6$

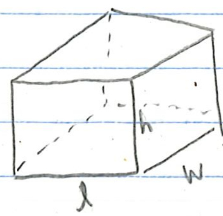
negative because its 25 ft OUT and the boat is being pulled IN

$\frac{d}{dt}(A^2 + B^2) = \frac{d}{dt}(C^2) \rightarrow 2A\frac{da}{dt} + 2B\frac{db}{dt} = 2C\frac{dc}{dt} \rightarrow 2(24)(\frac{dA}{dt}) + 2(7)(0) = 2(25)(-.6)$

$48\frac{dA}{dt} = -30 \qquad \frac{dA}{dt} = \frac{-30}{48} = \frac{-5}{8}$ ft/sec

When there is 25 ft of rope out, the boat is approaching the dock at 5/8 ft/sec.

8)



$V = \qquad \frac{dV}{dt} = ?$

$l = 8 \qquad \frac{dl}{dt} = .001 \text{ m/s}$

$W = 10 \qquad \frac{dw}{dt} = .001 \text{ m/s}$

$h = 3 \qquad \frac{dh}{dt} = .002 \text{ m/s}$

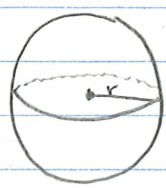
$\frac{d}{dt}(V = \underbrace{lwh}) \rightarrow \frac{dV}{dt} = \overbrace{lw\frac{dh}{dt} + h\left(l\frac{dw}{dt} + \frac{dl}{dt}w\right)}^{\text{Product rule}}$

$\frac{dV}{dt} = (8)(10)(.002) + (3)\left((8 \cdot .001) + (10 \cdot .001)\right)$

$= .214$ m^3/sec

The volume is increasing at a rate of $.214 \text{ m}^3/\text{sec}$

9)



Cross section →

$A = 144\pi \qquad \frac{dA}{dt} = 25$

$V = \frac{4}{3}\pi r^3 \qquad A = \pi r^2 \qquad 144\pi = \pi r^2 \quad \underline{r = 12}$

$r = 12 \qquad \frac{dr}{dt} = ?$

Find $\frac{dr}{dt}$, then use it to plug into $\frac{d}{dt}\left(V = \frac{4}{3}\pi r^3\right)$

$\frac{d}{dt}(A) = \frac{d}{dt}(\pi r^2) \rightarrow \frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right) = (25) = 2\pi(12)\frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = \frac{25}{24\pi}}$ mm^3/day

Now → $\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$

$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\frac{dr}{dt}\right) = \frac{4}{3}\pi\left(3(12)^2\left(\frac{25}{24\pi}\right)\right)$

$\frac{dV}{dt} = 600 \qquad$ The volume of the tumor is growing $600 \text{ mm}^3/\text{day}$

10 

They give us no value for r but $r = \frac{h}{2}$

$V =$ $\frac{dv}{dt} = -10$

$V = \frac{1}{3}\pi r^2 h$ $h = 10$ $\frac{dh}{dt} = ?$

$r = \frac{1}{2}h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$= \frac{1}{3}\pi \frac{1}{4} h^2 h$

$= \frac{1}{12}\pi h^3$

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{\pi}{12}h^3\right)$ $\frac{dv}{dt} = \frac{3\pi}{12}h^2\frac{dh}{dt}$

$-10 = \frac{3\pi}{12}(10)^2\frac{dh}{dt}$

dh/dt= $\frac{-2}{5\pi}$ cm/sec

The height of ice cream is decreasing at a rate of $\frac{2}{5\pi} \approx .127$ cm/sec