Printed Name: Key

Sample Exam II

MATH 231 – Calculus I

Instructions: This is an individual, closed-book, closed-notes exam. You may not give help, receive help, or discuss this exam with anyone. You may use a calculator or graphing calculator if you wish, but you may not use Maple, your cell phone, or any other electronic device.

Show work to get full credit (the correct answer may NOT be enough). Do all your work on the paper provided. You must include units in your answers whenever appropriate. Write clearly! Double check your answers!

Honor Pledge: As a member of the student body of Birmingham-Southern College, I realize my responsibility to the traditions of the institution, to my fellow students and to myself. I recognize the significance of the Honor System, and I pledge that I will not lie, cheat, or steal as a member of the Birmingham-Southern College community.

I have neither received nor given aid on this work, nor have I witnessed any such violation of the Honor Code.

Honor Pledge Signature:____________________

Helpful Formulas:

The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

The volume of a sphere is $V = \frac{4}{3}\pi r^3$ and has a surface area of $A = 4\pi r^2$.

The volume of a cylinder is $V = \pi r^2 h$ and has a surface area is $A = 2\pi r^2 + 2\pi r h$.

The area of a circle is $A = \pi r^2$ and the circumference of a circle is $C = 2\pi r$.

The Pythagorean Theorem is $a^2 + b^2 = c^2$ where a and b are the lengths of the sides of a right triangle while c is the length of the hypotenuse.

1. Calculate the derivative of the following functions. DO NOT SIMPLIFY. [25]

(a) $f(x) = \pi^{10} + \frac{2}{x^3} - \frac{1}{4}x^2 + e^x$ $= \pi^{10} + 2x^{-3} - \frac{1}{4}x^2 + e^x$

$f'(x) = 0 + 2(-3)x^{-4} - \frac{1}{4}(2x) + e^x$

(b) $f(x) = \frac{x^3 - 5}{x^{1/4} + 2x}$ $f'(x) = \frac{(x^{1/4}+2x)(3x^2) - (x^3-5)(\frac{1}{4}x^{-3/4}+2)}{(x^{1/4}+2x)^2}$

(c) $f(x) = 4\sqrt{3x(1-x^2)}$ $= 4(3x(1-x^2))^{1/2}$

$f'(x) = 4(1/2)(3x(1-x^2))^{-1/2}\left[3x(-2x) + (1-x^2)(3)\right]$

(d) $f(x) = \frac{e^{\sqrt{x^5-1}}}{2}$ $= \frac{1}{2}e^{\sqrt{x^5-1}} = \frac{1}{2}e^{(x^5-1)^{1/2}}$

$f'(x) = \frac{1}{2}e^{(x^5-1)^{1/2}}\left(\frac{1}{2}(x^5-1)^{-1/2}(5x^4)\right)$

(e) $f(x) = (3x+1)^4 e^{6x}$

$f'(x) = (3x+1)^4(e^{6x})(6) + e^{6x}(4)(3x+1)^3(3)$

2. Given the values in the table below, complete the following. [25]

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	4	2	2	4
1	3	3	-1	-2
2	-1	4	0	-1

(a) Find $\frac{d}{dx}\left(4f(x) - 3g(x)\right)$ at $x = 2$.

$= 4f'(x) - 3g'(x)$

At $x=2$: $4f'(2) - 3g'(2) = 4(4) - 3(-1) = 16 + 3 = 19$

(b) Find $\frac{d}{dx}\left(f(g(x))\right)$ at $x = 0$.

$= f'(g(x))\,g'(x)$

At $x=0$: $f'(g(0))\,g'(0) = f'(2)(4) = 4(4) = 16$

(c) Find $\frac{d}{dx}\left(2f(x) \cdot g(x)\right)$ at $x = 0$.

$= 2f(x) \cdot g'(x) + g(x)(2)f'(x)$

At $x=0$: $2f(0) \cdot g'(0) + g(0)(2)f'(0) = 2(4)(4) + 2(2)(2)$
$= 32 + 8 = 40$

(d) Find $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$ at $x = 1$.

$= \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

At $x = 1$: $\frac{g(1)f'(1) - f(1)g'(1)}{(g(1))^2} = \frac{(-1)(3) - 3(-2)}{(-1)^2}$
$= \frac{-3+6}{1}$
$= 3$

(e) Find $\frac{d}{dx}\left((f(x))^2\right)$ at $x = 2$.

$= 2(f(x))f'(x)$

At $x=2$: $2f(2)f'(2)$
$= 2(-1)(4)$
$= -8$

3. Consider the function defined implicitly by $x^2 - 3xy + y^2 = 1$

(a) Find $\dfrac{dy}{dx}$. [8]

$$\frac{d}{dx}\left[x^2 - 3xy + y^2\right] = \frac{d}{dx}[1]$$

$$2x - \left[3x\frac{dy}{dx} + y(3)\right] + 2y\frac{dy}{dx} = 0$$

$$2x - 3x\frac{dy}{dx} - 3y + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(-3x + 2y) = 3y - 2x$$

$$\frac{dy}{dx} = \frac{3y - 2x}{-3x + 2y}$$

(b) Find the equation of the tangent line to the function at the point $(3, 1)$. [6]

$$\frac{dy}{dx} \text{ at } (3,1): \quad \frac{3(1) - 2(3)}{-3(3) + 2(1)} = \frac{3-6}{-9+2} = \frac{-3}{-7} = \frac{3}{7}$$

Equation of tangent line at $(3,1)$: $y = \frac{3}{7}(x-3) + 1$.

4. Given the fact that $\dfrac{d}{dx}(\sin x) = \cos x$, compute the derivative of each of the following functions: [6]

(a) $y = x^5 \sin x$. Hint: Use product rule.

$$y' = x^5 \cdot \cos(x) + \sin(x)(5x^4)$$

(b) $y = \sin(x^5)$. Hint: Use chain rule.

$$y' = \cos(x^5)(5x^4)$$

5. A company's revenue, R in dollars, depends on the number of widgets w produced, i.e. $R = f(w)$. At the same time, the number of widgets produced depends on the amount of time, t measured in hours, their employees work each day, i.e. $w = g(t)$. At the end of the week, the company produces a report with the following data:

$$\underbrace{f(300)}_{\text{widgets}} = \underbrace{20,000}_{\text{dollars}} \qquad f'(\underbrace{300}_{\text{widgets}}) = \overbrace{125}^{\text{dollars/widget}} \qquad g(\underbrace{7}_{\text{daily hours}}) = \underbrace{300}_{\text{widgets}} \qquad g'(\overset{\text{daily hours}}{7}) = 10 \text{ widgets/daily hour}$$

(a) Explain what the value of $g'(7) = 10$ means in the context of the above scenario. [3]

$g'(7) = 10$ means that when the company's employees have worked 7 hours each day, they can produce 10 more widgets for an additional hour worked each day.

(b) Since $R = f(g(t))$, use the above information to find the value of $R'(7)$. [3]

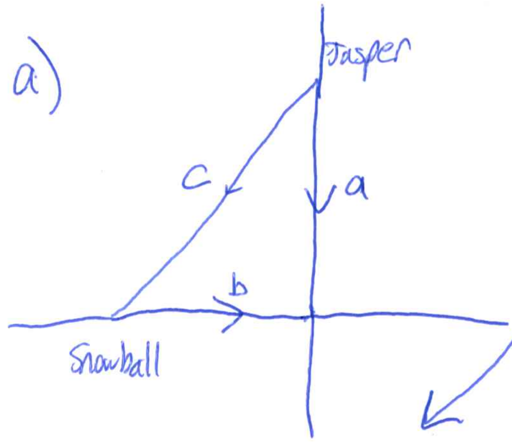
$$R'(t) = f'(g(t))\,g'(t)$$
$$R'(7) = f'(g(7))\,g'(7)$$
$$= f'(300)(10)$$
$$= 125(10)$$
$$= 1250$$

(c) Explain what the value of $R'(7)$ means in the context of the above scenario. [3]

$$R'(\underset{\text{hour}}{7}) = \underset{\text{dollars/hour}}{1250}$$

$R'(7) = 1250$ means that when the company employees work 7 hours per day, their revenue increases by 1250 dollars for each additional hour per day they work

6 a)



$a = 12$ $\quad \frac{da}{dt} = -3$

$b = ?\ 5$ $\quad \frac{db}{dt} = -2$

$c = 13$ $\quad \frac{dc}{dt} = \star$

Find b: $12^2 + b^2 = 13^2 \Rightarrow 144 + b^2 = 169 \Rightarrow b^2 = 25 \Rightarrow b = 5$

b) $a^2 + b^2 = c^2$

$$\frac{d}{dt}(a^2 + b^2) = \frac{d}{dt}(c^2)$$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

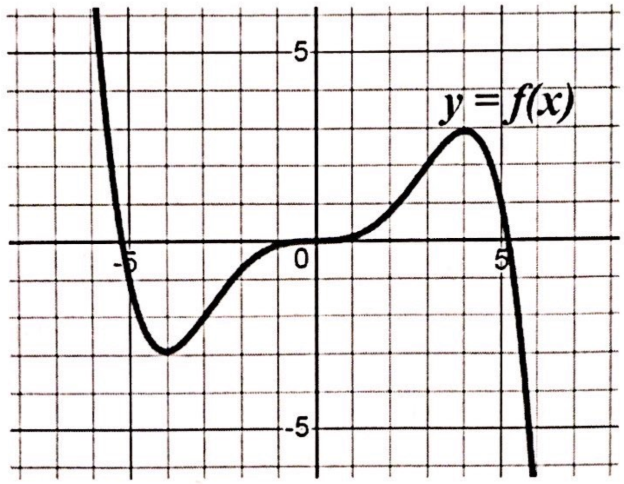
$$2(12)(-3) + 2(5)(-2) = 2(13)\frac{dc}{dt}$$

$$\frac{-92}{26} = \frac{26\frac{dc}{dt}}{26}$$

$$\frac{dc}{dt} = \frac{-46}{13} \approx -3.538$$

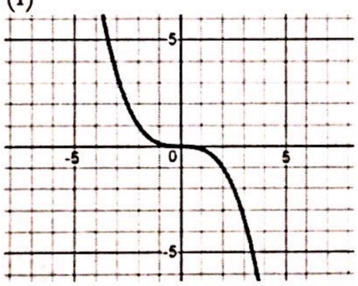
c) When Jasper and Snowball are 13m apart, the distance between them is decreasing by $\frac{46}{13} \approx 3.538$ m/s.

7. The graph of $y = f(x)$ is given below.

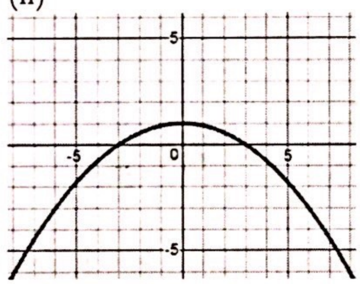


(a) From the graph of $y = f(x)$ above, circle the correct graph of $y = f'(x)$ from the choices below. [2]

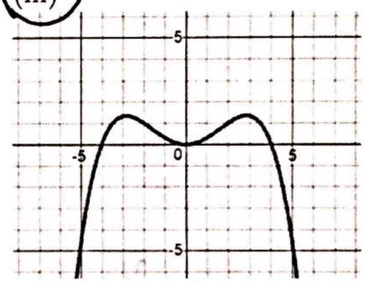
(i)



(ii)



(iii)



(b) Briefly describe why that is the correct graph. [4]

$f(x)$ is decreasing on $(-\infty,-4)$, so $f'(x) < 0$ on $(-\infty,-4)$ (i.e. $f'(x)$ is below the x-axis). Then, $f(x)$ increases on $(-4,4)$ so $f'(x) > 0$ on that interval. Finally, $f'(x)$ decreases on $(4,\infty)$ so $f'(x) < 0$ on $(4,\infty)$.