**MA 209 Lab 10**

**Normal Distributions and the Empirical Rule**

1. At a certain community college, pick a student at random and record their age,  The distribution is approximately normal with a mean of 33 years and a standard deviation of 5 years.
   1. Is the random variable age categorical or quantitative? If quantitative, is it discrete or continuous?

It is quantitative continuous.

* 1. Pick one individual student at random. Find the z-score and use the Empirical Rule to find the probability that this student is younger than 43. Shade a region under a labeled normal curve to represent your answer.

. The area to the left of the point that’s two standard deviations higher than the mean includes the middle 95% plus the 2.5% in the left tail. Thus the total probability is 0.975.

* 1. Pick one individual student at random. Find the z-score and use the Empirical Rule to find the probability that this student is older than 28. Shade a region under a labeled normal curve to represent your answer.

. The area to the left of the point that’s one standard deviation lower than the mean includes the left tail that is outside the middle 68%. Thus the total probability is ½ of 32%, or 0.16.

* 1. Pick one individual student at random. Use the Empirical Rule to find the probability that this student is between 28 and 43. Shade a region under a labeled normal curve to represent your answer.

The area between the two is equal to , which is the area to the left of 2 minus the area to the left of -1. Thus the total is 0.975-0.16 = 0.815.

* 1. Pick one individual student at random. Find the probability that this student is between 33 and 40. Shade a region under a labeled normal curve to represent your answer.

The probability will be equal to , but we need Python for this one or some other technology. The result is

* 1. Pick one individual student at random. Find the probability that this student is older than 53.
  2. Pick one individual student at random. Find the probability that this student is exactly 33.

This probability is 0 because the area under a curver over a single point is always 0.

* 1. What percentile is a 38-year-old student in?

The percentile is the percentage of observations less than or equal to a given value. Since such a student is in the 84th percentile.

* 1. How old is a student is in the 90th percentile?

Here we need to find the such that Using Python we find

**Sampling Distributions and the Central Limit Theorem**

1. Suppose that, at a certain community college, the population distribution of the ages of the students  is approximately normal with a mean of 33 years and a standard deviation of 5 years.
   1. What is the mean of the sampling distribution of sample means  when the samples are of size  Use the correct notation (the appropriate letter) and include units in your answer.

By the CLT, the mean of the sampling distribution is the same as the population mean since the sample size is larger than 30 (and in this case the population is roughly normal to begin with). Thus,

* 1. What is the standard deviation of the sampling distribution of sample means when the samples are of size 100? Use the correct notation (the appropriate letter) and include units in your answer.

By the CLT, the standard deviation of the sampling distribution is since the sample size is larger than 30 (and in this case the population is roughly normal to begin with). Thus,

* 1. Suppose that you take a SRS of 100 students. What is the probability that the sample mean of this particular sample is between 32 and 34? Shade a region under a labeled normal curve to represent your answer.

Since , by the Empirical Rule.

* 1. Suppose that you take a SRS of 100 students. What is the probability the sample mean of this particular sample is greater than 34.5 years? Shade a region under a labeled normal curve to represent your answer.

Since , by the Empirical Rule.

* 1. Suppose that you take a SRS of 100 students. Approximately what is the probability that the sample mean of this particular sample is greater than 38?

Since , .

* 1. Pick one individual student at random. Find the probability that this one particular student is older than 38. Shade a region under a labeled normal curve to represent your answer.

Since , by the Empirical Rule.

* 1. Are your answers in part (e) and part (f) the same? Explain why or why not by describing whether there is the same variability, more variability, or less variability in the distribution of sample means compared to the distribution of the original population of students.

The probability in part e. is much smaller than in part f. because averages are less variable than single observations. If we take a SRS of size 100, a sample of size 100 comes from a distribution with standard deviation 0.5, which is much less than the standard deviation for a single observation, which is 5.

* 1. Find  and . Which is higher and why?

by the Empirical Rule.

. The first one is much higher because sample means have much less variation than single observations.

1. Suppose that 80% of cell phone charging cables work properly. Suppose we take random samples of size 100. The applet below shows the population distribution with a population proportion of  and a data distribution from just one sample that was drawn that happened to have a sample proportion of  and the *sampling distribution* of the sample proportion which you can see is approximately normal (the sampling distribution is actually the distribution of all possible samples, but the illustration has 90,000 samples drawn).



* 1. What is the mean of the sampling distribution of the sample proportion  when the samples are of size  Use the correct notation (the appropriate letter).

By the CLT, the mean for the distribution is the same as the population proportion, , which is 0.8. Thus

* 1. What is the standard deviation of the sampling distribution of the sample proportion  when the samples are of size 100? Use the correct notation (the appropriate letter).

By the CLT, the standard deviation for the distribution equals , so

* 1. Suppose that you take a **S**imple **R**andom **S**ample of size 100. What is the probability that the sample proportion is less than 0.75? Shade a region under a labeled normal curve to represent your answer.

Since ,

* 1. Suppose that you take a SRS of size 100. What is the probability that the sample proportion is greater than 0.85? Shade a region under a labeled normal curve to represent your answer.

Since ,

* 1. Suppose that you take a SRS of size 100. What is the probability that the sample proportion is more extreme than the sample we got (meaning the sample proportion is less than 0.75 or greater than 0.85)? Shade a region under a labeled normal curve to represent your answer.

We need

* 1. Find , , and 

Using Python, we get 0.988, 0.012, and 0.