Classical Mechanics
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Chapter 5 Problems Damped Oscillations & Resonance

Bonnie Flood light .com

5.42) A large Foucalt pendulum such as hangs in many science museums can swing for many hours before it damps out. Taking the decay time to be about 8 hours and the length to be 30 meters, find the quality factor Q.

5.33) The solution for $x(t)$ for a driven underdamped oscillator is most conveniently found in the form

$$x(t) = A\cos(\omega t - \varphi) + e^{-\beta t}\left(B_1 \cos(\omega_1 t) + B_2 \sin(\omega_2 t)\right)$$

[The constants A and φ are determined by

$$A^2 = \frac{F_0^2}{(\omega_0^2 - \omega^2)^2 + 4c^2\omega^2}$$

and $$\varphi = \arctan \frac{2c\omega}{(\omega_0^2 - \omega^2)}$$] - verify the forms

$B_1 = x_0 - A\cos\varphi$ and $B_2 = \frac{1}{\omega_1}(v_0 - \omega A \sin\varphi + \beta B_1)$

5.27) As the damping on an oscillator is increased there comes a point when the name "oscillator" can never pass through the origin, $x = 0$ more than once a.) To prove this, prove that a critically-damped oscillator can never pass through the origin, $x=0$ more than once. b) prove the same for an overdamped oscillator.

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Remark: About notation -- Taylor uses β as a damping constant/parameter instead of 'c'. He defines $\frac{b}{m}=2\beta$ to simplify the differential equation below
$m\ddot{x}+b\dot{x}+kx=0$
he also defines $\omega_0=\sqrt{\frac{k}{m}}$ to
Lastly, instead of using q or γ, he uses 'r'. These are all dummy variables, to be sure.

$\omega_0 \equiv$ natural frequency

Assume that the variables in the equation of motion for a damped oscillator are written as $\ddot{x}+2\beta\dot{x}+\omega_0^2x=0$. ($\beta$ and ω_0 are inverse time parameters) And that the auxiliary equation is expressed as $q^2+2\beta q+\omega_0^2=0$; solving, we obtain

$$q=\frac{-2\beta\pm\sqrt{(2\beta)^2-4(1)(\omega_0^2)}}{2}$$

$$=\frac{-2\beta\pm2\sqrt{\beta^2-\omega_0^2}}{2}$$

$$=-\beta\pm\sqrt{\beta^2-\omega_0^2}$$

5.20

Verify that the decay parameter $\beta-\sqrt{\beta^2-\omega_0^2}$ for an overdamped oscillator ($\beta>\omega_0$) decreases with increasing β. Sketch its behavior for $\omega_0<\beta<\infty$

5.21 Verify that the function $x(t)=te^{-\beta t}$ is indeed a second solution of the equation of motion

$$\ddot{x}+2\beta\dot{x}+\omega_0^2x=0$$

for a critically-damped oscillator ($\beta=\omega_0$)

5.22. a.) Consider a cart on a spring which is critically-damped. At time $t=0$, it is sitting at its equilibrium position and is kicked in the positive direction with velocity v_0. Find its position $x(t)$ for all subsequent times and sketch your answer.

b.) Do the same for the case that it is released from rest at position $x=x_0$. In this latter case, how far is the cart from equilibrium position after a time period to $\tau_0=2\pi/\omega_0$, the period in the absence of any damping.