Ex: Sketch the graph $f(x) = \frac{1}{x^2-1} = (x^2-1)^{-1}$

Sol: (a) Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

(b) 1st deriv. info:

$$f'(x) = -1(x^2-1)^{-2}(2x)$$
$$= \frac{-2x}{(x^2-1)^2}$$
$$0 = \frac{-2x}{(x^2-1)^2}$$
$$0 = -2x$$
$$x = 0 \leftarrow \text{CN}$$

$f'(x)$ undefined at $x = -1, 1$

but NOT CN's because not in domain of f.

Interval	sign of f'	conclusion for f
$(-\infty, -1)$	+	increasing
$(-1, 0)$	+	increasing
$(0, 1)$	−	decreasing
$(1, \infty)$	−	decreasing

local max at $x=0$: $(0, f(0)) = (0, -1)$

(c) 2nd deriv info:

$$f''(x) = \frac{(x^2-1)^2(-2) - (-2x)(2)(x^2-1)(2x)}{((x^2-1)^2)^2}$$
$$= \frac{-2(x^2-1)^2 + 8x^2(x^2-1)}{(x^2-1)^4}$$
$$= \ldots$$
$$= \frac{6x^2+2}{(x^2-1)^3}$$

$$0 = \frac{6x^2+2}{(x^2-1)^3}$$
$$0 = 6x^2+2$$
$$-2 = 6x^2$$
$$\frac{-2}{6} = x^2$$

NO solution

$f''(x)$ undefined at $x = -1, 1$ but NOT in domain of f.

Interval	sign of f''	conclusion for f
$(-\infty,-1)$	+	concave up
Ⓓ $(-1,1)$	−	concave down
Ⓓ $(1,\infty)$	+	concave up

(d) Asymptotes:

horizontal: $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1}{x^2-1} = 0$

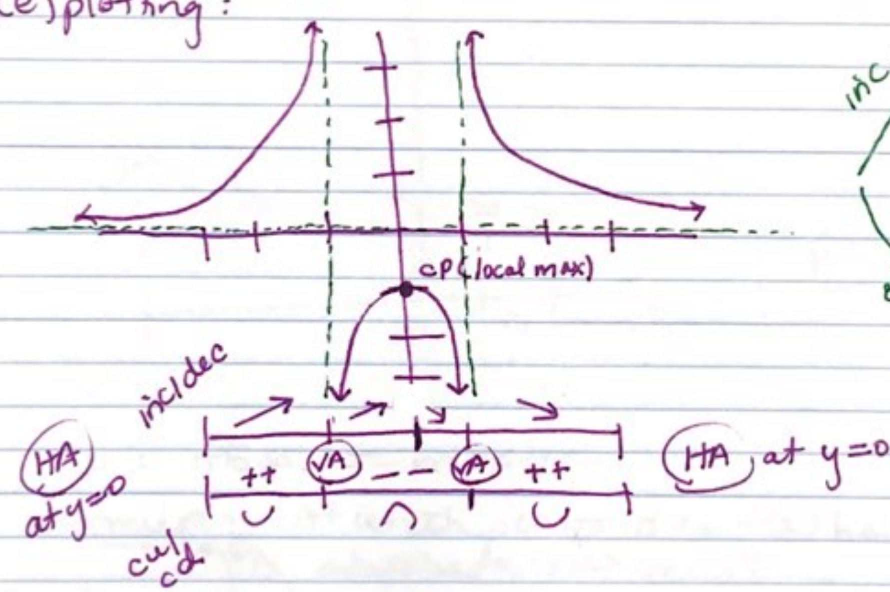
$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{1}{x^2-1} = 0$

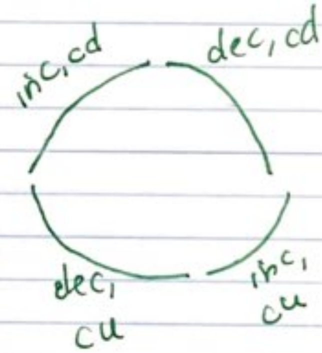
vertical: $\lim_{x\to a} f(x) = \text{“}\frac{\text{nonzero}}{\text{zero}}\text{”}$

$\lim_{x\to 1} \frac{1}{x^2-1} = \frac{1}{0}$

$\lim_{x\to -1} \frac{1}{x^2-1} = \frac{1}{0}$

(e) plotting:





Ex: Sketch a graph of a function w/ these properties:

a) $f(x) = 0$ at $x = 1$ point $(1, 0)$

b) $f'(1) = 0$ and $f''(1) = 0 \rightarrow$ possible IP at $x = 1$

↳ $x = 1$ is a CN (in domain from (a))

c) $\lim_{x \to 0} f(x) = \infty$ ↳ VA at $x = 0$, $\lim_{x \to 2} f(x) = -\infty$ ↳ VA at $x = 2$

f decreasing $(0, 2)$

d) $f'(x) > 0$ when $x < 0$ and $x > 2$ f increasing $(-\infty, 0) \cup (2, \infty)$

e) $f'(x) < 0$ when $0 < x < 2$

f) $f''(x) < 0$ when $1 < x < 2$, $x > 2$

f concave down $(1, 2) \cup (2, \infty)$

g) $\lim_{x \to \infty} f(x) = 1$ and $\lim_{x \to -\infty} = 0$.

HA at $y = 1$ HA at $y = 0$