

Worksheet 8 KEY - #1

a) $a = -1$ $b = 4$ integrand $= (3-2x)$

b) $\Delta x = \frac{4 - (-1)}{5} = 1$

Divide interval into $n=5$ subunits of $\Delta x=1$

$f(x_0) = f(a) = f(-1) = 5$ $f(x_1) = f(0) = 3$ $f(x_2) = f(1) = 1$

$f(x_3) = f(2) = -1$ $f(x_4) = f(3) = -3$

Sum: $(1)(5+3+1-1-3) = 5$

c) $\Delta x = \frac{4 - (-1)}{n} = \frac{5}{n}$

d) $x_k = -1 + k\left(\frac{5}{n}\right)$

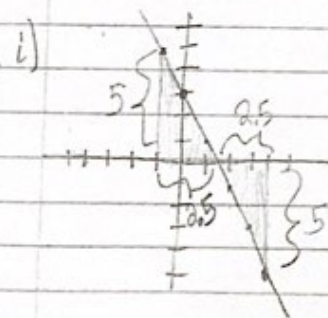
e) $f(x_k) = \left(3 - 2\left(-1 + \frac{5k}{n}\right)\right)$
 $= 3 + 2 - \frac{10k}{n}$

$= 5 - \frac{10k}{n}$

f) $\sum_{k=1}^n f(x_k) \Delta x = \sum_{k=1}^n \left(5 - \frac{10k}{n}\right) \left(\frac{5}{n}\right) = \sum_{k=1}^n \frac{25}{n} - \frac{50k}{n^2}$
 $= \sum_{k=1}^n \frac{25}{n} - \frac{50}{n^2} \sum_{k=1}^n k = \left(\frac{25}{n}\right) \cdot n - \left(\frac{50}{n^2}\right) \left(\frac{n(n+1)}{2}\right)$
 $= 25 - \left(\frac{25n+25}{n}\right)$

g) $\sum_{k=1}^{\infty} f(x_k) \Delta x = 25 - \left(\frac{25(100)+25}{100}\right) \approx \boxed{-0.25}$

h) $\int_{-1}^4 (3-2x) dx = \lim_{n \rightarrow \infty} \left(25 - \left(\frac{25n+25}{n}\right)\right) = 25 - \left(\frac{25}{1}\right) = \boxed{0}$



$\int_{-1}^4 f(x) dx = \frac{1}{2} (2.5)(5) - \frac{1}{2} (2.5)(5) = \boxed{0}$
 ↑ above x-axis ↑ below x-axis