KEY

Sample Exam IV

Directions: You have one hour to complete this exam. It is due at the end of the hour in your instructor's office. You may use your calculators, but you may not use any books or notes. Also, you must work alone.

I, ___KEY___, am fully aware of and have abided by the BSC Honor Code in completing this exam.

Please work in the space provided and *show all of your work.*

The following summation formulas may be useful:

$$\sum_{k=1}^{n} c = cn.$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$

1. (6 pts) State the Fundamental Theorem of Calculus:

If f is continuous on $a \leq x \leq b$, and $F'(x) = f(x)$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

2. (5 pts each) Calculate the following:

(a) $\int \frac{3}{x^2} + 4\sqrt{x}\, dx$ $= \int 3x^{-2} + 4x^{1/2}\,dx$

$$= \frac{3x^{-1}}{-1} + \frac{4x^{3/2}}{3/2} + C$$

(b) $\int \frac{1}{x} + \ln(\pi)\pi^x + 12\, dx$

$$= \ln|x| + \pi^x + 12x + C$$

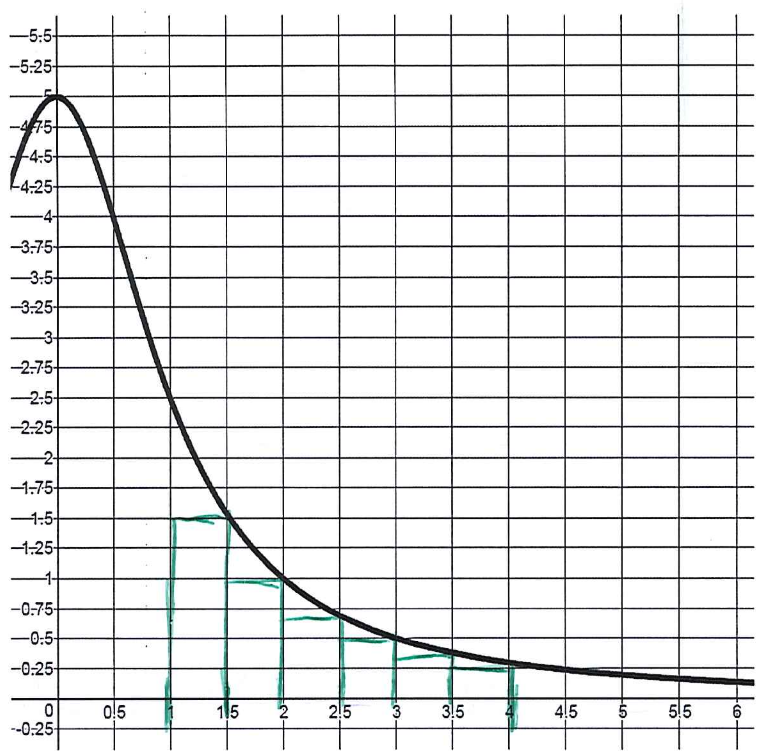
(c) $\int_0^{\pi/2} \cos(x)\, dx$

$$= \sin x \Big|_0^{\pi/2} = \sin\left(\frac{\pi}{2}\right) - \sin(0) = 1 - 0 = 1$$

(d) $\int_1^4 \frac{1}{\sqrt{x}}\, dx$ $= \int_1^4 x^{-1/2}\,dx = \frac{x^{1/2}}{1/2}\Big|_1^4 = 2\sqrt{x}\Big|_1^4 = 2\sqrt{4} - 2\sqrt{1} = 2(2) - 2(1) = 2$

3. The graph of $f(x)$ is given below.

(a) (6 pts) On the graph below, draw the rectangles corresponding to a **Right-Hand Riemann Sum** with **six** rectangles for the integral $\int_1^4 f(x)\, dx$.



(b) (4 pts) Estimate the value of the integral using a Right-Hand Riemann Sum with **six** rectangles. $\Delta x = \frac{4-1}{6} = \frac{1}{2}$

$$\tfrac{1}{2}\cdot f(1.5) + \tfrac{1}{2}\cdot f(2) + \tfrac{1}{2}\cdot f(2.5) + \tfrac{1}{2} f(3) + \tfrac{1}{2} f(3.5) + \tfrac{1}{2} f(4)$$

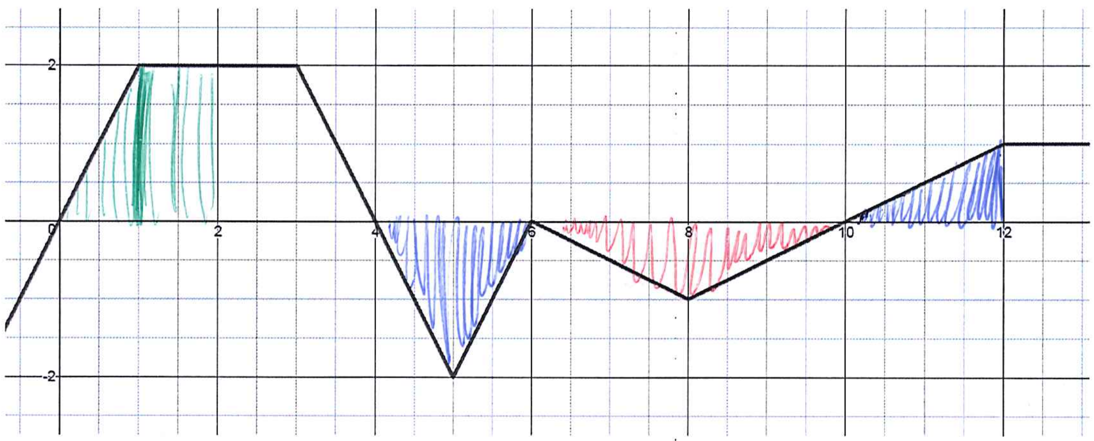
$$= \tfrac{1}{2}\left[1.5 + 1 + .7 + .5 + .35 + .25\right]$$

$$= \tfrac{1}{2}[4.3] = 2.15$$

(c) (3 pts) Is the estimate an over-estimate or an under-estimate? Why?

underestimate. The right endpoints produce lower y-values on each subinterval.

4. (4 pts each) The graph of $f(x)$ is given below. Calculate the following:



$$\int_0^2 f(x)\,dx = \frac{1}{2}(1)(2) + (1)(2) = 3$$

$$\int_{12}^{10} f(x)\,dx = -\int_{10}^{12} f(x)\,dx = -\frac{1}{2}(2)(1) = -1$$

$$\int_4^6 f(x)\,dx = -\frac{1}{2}(2)(2) = -2$$

$$\int_6^{10} 2\,f(x)\,dx = 2\int_6^{10} f(x)\,dx = 2\left(-\frac{1}{2}(4)(1)\right) = -4$$

5. Let $R(t)$ be the growth rate of plankton measured in kg/day in a certain region off the Alaskan coast in **June** where $t = 0$ corresponds to **June 1st**. Suppose that on **July 1st** ($t = 30$), that region held a biomass of 345,000 kg of plankton.

(a) (4 pts) What does the statement $\int_0^{30} R(t)\, dt = 134,000$ mean in English?

Between June 1st and July 1st, the biomass of plankton grew by 134,000 kg.

(b) (4 pts) Find the amount of Plankton in that region on **June 1st.**

~~[illegible]~~ On July 1st, there was 345,000 kg so on June 1st, the biomass would have been 345,000 - 134,000 = 211,000 kg.

6. While driving at night you see a deer ahead in the road and slam on your breaks to decelerate (so that you are always slowing down). The 'black box' in your car records the following data based on the speedometer where $t = 0$ is the time at which your foot hits the break pedal.

Time (in seconds)	0	2	4	6	8
Speed (in ft/second)	50	30	10	5	0

(a) (4 pts) Give an over-estimate for the distance traveled over the 8 second period.

$(2)(50) + (2)(30) + (2)(10) + (2)(5) = 190$ ft.

(b) (4 pts) Give an under-estimate for the distance traveled over the 8 second period.

$(2)(30) + (2)(10) + (2)(5) + (2)(0) = 90$ ft.

(c) (3 pts) Suppose the deer was 200 feet ahead of you when you first saw it and slammed on the breaks. Did you hit the deer? How do you know?

No because the maximum distance I could have travelled was 190 ft since that was an overestimate. Therefore, I would have stopped at least 10 ft. short of the deer.

7. For the integral $\int_0^4 3x^2\,dx$ do the following:

(a) (6 pts) Find the Right-hand Riemann Sum for n rectangles. Simplify.

$$\Delta x = \frac{4-0}{n} = \frac{4}{n}$$

$$x_k = 0 + k\left(\frac{4}{n}\right) = \frac{4k}{n}$$

$$f(x_k) = 3\left(\frac{4k}{n}\right)^2 = 3\left(\frac{16k^2}{n^2}\right) = \frac{48k^2}{n^2}$$

$$\sum_{k=1}^{n}\left(\frac{4}{n}\right)\left(\frac{48k^2}{n^2}\right) = \sum_{k=1}^{n}\frac{192k^2}{n^3} = \frac{192}{n^3}\cdot\sum_{k=1}^{n}k^2$$

$$= \frac{192}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} = \frac{192}{n^3}\cdot\frac{2n^3+3n^2+n}{6} = \frac{384n^3+576n^2+192n}{6n^3}$$

(b) (2 pts) Using your formula above, find the Right-hand Riemann Sum for $n = 100$ rectangles. Simplify your answer as much as you can (i.e. I don't want to see a summation symbol). You should have a number.

$$\frac{384(100^3)+576(100^2)+192(100)}{6(100^3)} = 64.9632$$

(c) (4 pts) Calculate the exact value of the integral by taking the limit as n goes to infinity of your formula in part (a).

$$\lim_{n\to\infty}\frac{384n^3+576n^2+192n}{6n^3} = \frac{384}{6} = 64$$

(d) (6 pts) Verify your answer using the Fundamental Theorem of Calculus.

$$\int_0^4 3x^2\,dx = \frac{3x^3}{3}\Big|_0^4 = x^3\Big|_0^4 = 4^3 - 0^3 = 64 - 0 = 64$$