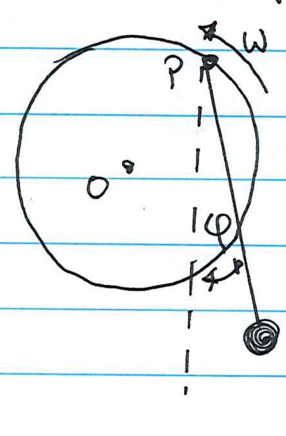
5-4-2022

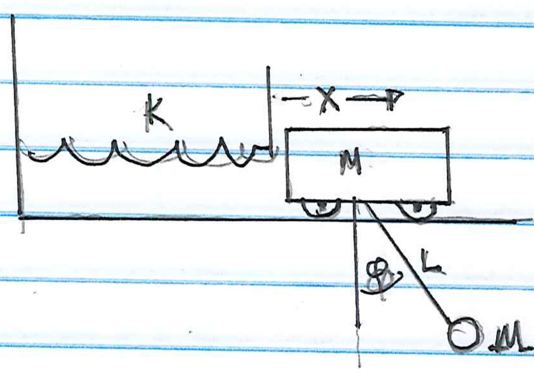
7.22) Using the angle φ as a generalized coordinate, write down the Lagrangian for a simple pendulum of length ℓ suspended from the ceiling of an elevator that is accelerating upward with constant acceleration a. (Be careful when writing 'T'; it is probably safest to write the bob's velocity in component form.) Find the Lagrangian equation of motion and show that it is the same as that for a normal, nonaccelerating pendulum, except that g has been replaced by $g+a$. In particular, the angular frequency of small oscillations is $\sqrt{(g+a)/\ell}$

7.29) The figure shows a simple pendulum (mass m length ℓ) whose point of support P is attached to the edge of a wheel (center O radius R) that is forced to rotate at a fixed angular velocity ω. At $t=0$, the point P is level with O on the right. Write down the Lagrangian and find the equation of motion for the angle φ. [Hint: Be careful writing down the kinetic energy, E_K. A safe way to get the velocity right is to write the position of the bob at time t, and then differentiate.] Check that your answer makes sense in the special case that $\omega=0$.



5-4-2022

7.31) A simple pendulum (mass M and length l) is suspended from a cart of mass m that can oscillate at the end of a spring of force constant k, as shown in the figure. a.) Write the Lagrangian in terms of the 2 generalized coordinates x and φ, where x is the extension of the spring from its equilibrium length (Read the hint in problem 7.29). Find the 2 Lagrange equations. (Warning: They're pretty nasty!) b.) Simplify the equations in the case that both x and φ are small. (Note, they're still pretty nasty; in particular, they are still coupled in that each equation involves both variables).



8.2) The splitting of the Lagrangian into 2 independent pieces, $\mathcal{L} = \mathcal{L}_{cm} + \mathcal{L}_{rel}$ extends easily to more general situations. To illustrate this, consider 2 masses m_1 and m_2 in a uniform gravitational field $\vec{g}$ which interact via a potential energy $U(r)$. a.) Show that the Lagrangian can be decomposed to $\mathcal{L} = \mathcal{L}_{cm} + \mathcal{L}_{rel}$ (8.13) b.) Write down Lagrange's equations for the 3 CM coordinates X, Y, Z and describe the motion of the CM. Write down the three Lagrange equations for the relative coordinates and show clearly that the motion of $\vec{r}$ is the same as a single particle of mass equal to the reduced mass μ with position $\vec{r}$ and potential energy $U(r)$.

5-4-2022

Lagrangian - Applications
(center of mass and reduced mass)

8.3) Two particles of masses m_1 and m_2 are joined by a massless spring of natural length L and force constant k. Initially, m_2 is resting on a table and m_1 is being held vertically above m_2 at height L. At time $t=0$, m_1 is projected vertically upward with initial velocity v_0. Find the positions of the two masses at any subsequent time t (before either mass returns to the table and describe the motion. (Assume that v_0 is small enough that the 2 masses don't collide).

8.4) Using the Lagrangian $\mathcal{L} = \mathcal{L}_{cm} + \mathcal{L}_{rel}$ write down the 3 Lagrangian equations for the relative coordinates x, y, z and show clearly that the motion of the relative position $\vec{r}$ is the same as that of a single particle with position $\vec{r}$, potential energy $U(r)$, and masses equal to the reduced mass μ.

8.5) The momentum $\vec{p}$ conjugate to the relative position $\vec{r}$ is defined with components $p_x = \frac{\partial \mathcal{L}}{\partial \dot{x}}$ and so on. Prove that $\vec{p} = \mu \dot{\vec{r}}$. Prove also that in the CM frame $\vec{p}$ is the same as $\vec{p}_1$, the momentum of particle 1 (and also $-\vec{p}_2$).

8.6) Show that in the CM frame, the angular momentum $\vec{l}_1$ of particle 1 is relative to the total angular momentum $\vec{L}$ by $l_1 = \left(\frac{m_2}{M}\right)\vec{L}$ and likewise $l_2 = \left(\frac{m_1}{m_2}\right)\vec{L}$. Since $\vec{L}$ is conserved, this shows that the same is true of $\vec{l}_1$ and $\vec{l}_2$ separately in the CM frame.

3-2-2022

Lagrangian - Applications

7.23 A small cart of mass m is mounted on rails inside a large cart. The two are attached by a spring (force constant k) in such a way that the small cart is in equilibrium at the midpoint of the large. The distance of the small cart from its equilibrium is denoted by x and that of the large one from a fixed point on the ground is X, as shown in the figure. The large cart is now forced to oscillate such that $X = A\cos\omega t$, with both A and ω fixed. Set up the Lagrangian for the motion of the small cart and show that the Lagrange equation has the form $\ddot{x} + \omega_0^2 x = B\cos\omega t$

where ω_0 is the natural frequency $\omega_0 = \sqrt{\frac{k}{m}}$ and B is a constant. [This is the same form assumed in section 5.5, Equation 5.57, for driven oscillations (except that here we are ignoring damping).