13.11 Measure x along the tracks in the direction of travel, y crossways, and z vertically up, all three relative to the car. The ball's velocity relative to the ground is $(V+\dot{x}, \dot{y}, \dot{z})$, so the Lagrangian is $\mathcal{L} = E_K - U =$

$$\mathcal{L} = \tfrac{1}{2}m\left[(V+\dot{x})^2 + \dot{y}^2 + \dot{z}^2\right] - mgz.$$

The generalised momentum has components

$$p_x = \frac{\partial \mathcal{L}}{\partial \dot{x}} = m(V+\dot{x}), \quad p_y = m\dot{y}, \quad p_z = m\dot{z}$$

Notice that the generalized momentum is the momentum relative to the ground, not to the moving car! We can solve for $\dot{x} = (p_x - mV)/m$, so that

$$\mathcal{H} = p_x\dot{x} + p_y\dot{y} + p_z\dot{z} - \mathcal{L} = \frac{p^2}{2m} - p_x V + mgz$$

From here, we can derive the expected equation of motion, but the point is here is to note that $\mathcal{H}$ is not equal to the energy $E_K + U$ (neither relative to the car nor relative to the ground), because

$$(E_K+U)_{\text{rel. to car}} = \tfrac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2) + mgz$$

$$= \frac{p^2}{2m} - p_x V + \tfrac{1}{2}mV^2 + mgz \neq \mathcal{H}$$

and $(E_K+U)_{\text{rel. to ground}} = \frac{p^2}{2m} + mgz \neq \mathcal{H}$