Temperature and Thermal Expansion

We begin the new semester by studying **thermodynamics**, the study of temperature, heat, and how heat can be used to do work.

Temperature and Heat

The first thing we need to do is to distinguish between two different, but often confused, concepts. Their precise definition is subtle and highly mathematical. However, we can distinguish them using reasonable operational definitions. Not to be sarcastic, be we can operationally define **temperature** as the quantity a thermometer measures. The operational definition will become clearer below. **Heat**, on the other hand, is the energy transferred between two objects *solely because they have different temperatures*.

We say that heat always *spontaneously* transfers ("flows") from a high temperature object to a low temperature object when they are in thermal contact. **Spontaneously** basically means "when the objects are left alone," or in the absence of some work being done by a clever device like a refrigerator. If you put a hot stone in cold water, the stone will cool down and the water will warm up. That is because the stone loses energy, and the water gains the same amount of energy, as heat is transferred from the stone to the water. Eventually, the stone and water will reach the same temperature. At that point we say the two objects are in **thermal equilibrium**: no heat will flow between them.

The concept of thermal equilibrium allows us to reconsider our operational definition of temperature. This comes from something called the **Zeroth Law of Thermodynamics**: when two objects are each in thermal equilibrium with a third object, then they are in thermal equilibrium with each other. This means we can associate with each object a number, called "temperature," that is the same for two objects when they are in thermal equilibrium. At this point you can be assured that if objects A and B are at the same temperature, and objects B and C are at the same temperature, then objects A and C are at the same temperature. You know

this even if you never bring objects A and C into thermal contact to determine whether or not heat would actually flow between them! This is the basis for how a thermometer can work. The thermometer (object B) is calibrated against a standard in the factory (object A). Therefore, when you put the thermometer into thermal equilibrium with your body (object C), you can be sure that your body temperature is measured accurately against the standard.

Temperature Scales

You may be familiar with several different temperature scales in common use. The most popular scale in the US is the **Fahrenheit** scale, which is useful for measuring outside temperatures. Indeed, the most common temperatures seen in the US tend to fall in the range 0°F–100°F. However, the Fahrenheit scale is not particularly useful for scientific applications.

The **Celsius** scale, on the other hand, is very useful for scientific work, particularly chemistry. This scale is anchored to thermal properties of water at atmospheric pressure: water freezes at 0°C and boils at 100°C.

These two scales are easily related by conversion formulas, but you can always re-derive them because the scales are linearly related. The key point is that a temperature difference of 5°C corresponds to a temperature difference of 9°F. The temperature for freezing water is 32°F and 0°C, so we can find the Fahrenheit temperature, T_F, corresponding to a given Celsius temperature, T_C, according to the formula

$$T_F = \frac{9}{5} T_C + 32 \ .$$

It is easy to invert the formula to get

$$T_C = \frac{5}{9} (T_F - 32) \ .$$

Interestingly, if you convert $T_F = 98.6$°F in to Celsius, you get a human body temperature of 37°C. This hints that the average human body temperature was originally determined on the Celsius scale, and rounded to two significant digits. When you convert this to Fahrenheit, you get a temperature between 98°F and 99°F. In other words, the commonly accepted average body temperature we know as 98.6°F probably is specified to one more significant digit than is actually justified.

If you find yourself traveling anywhere else in the world, how can you determine from the weather report how to dress? Here’s a simple table:

Celsius	Fahrenheit	How It Feels
40°C	104°F	Very hot
30°C	86°F	Hot
20°C	68°F	Cool room temperature
10°C	50°F	Cool / Jacket weather
0°C	32°F	Freezing

You can also use a simple mnemonic for Celsius temperatures: “30 is hot, 20 is nice, 10 is cool, 0 is ice.”

An even more useful temperature scale for science, and the most important one for physics, is the **Kelvin** scale. This scale is defined such that a temperature difference of 1°C corresponds to a temperature difference of 1 K, and that 0 K corresponds to **absolute zero**. At 0 K all random molecular motion would cease and a gas would exert no pressure on the walls of its container. According to the **third law of thermodynamics**, you can never have an object at absolute zero, but it is an important limit of how low temperatures can be.

The conversion between Celsius and Kelvin is simple: $T_K = T_C + 273$, or $T_C = T_K - 273$. It is useful to remember that room temperature is in the range 295 K–300 K (approximately 71°F–80°F).

For applications in which the *difference* in temperatures is important, such as linear expansion and heat conduction, it doesn’t matter whether you use Kelvin or Celsius (or even Fahrenheit) becasue in these cases it does not matter where the zero of the temperature scale is defined. By anaology, you may weigh yourself clothed at the gym, and unclothed at home, but both scales should accurately tell you how much weight you lost in the past week.

For applications in which the temperature figures *directly*, such as the ideal gas law and the power radiated by a star, it is the absolute temperature that is important. For such cases you must always use Kelvin. (Actually, you can also use the **Rankine** scale, an absolute temperature scale related to Fahrenheit.)

Thermal Expansion

The thermometer above works on the principle of **thermal expansion**. For most substances, as the temperature of an object increases, it will increase in size. You see this when the mercury in the thermometer fills the tube more as you warm it in your mouth. You also see it when

you drive across bridges and notice *expansion joints*, which are gaps in the road that fill as the bridge expands in hot weather.

The important thing to remember is that the amount of expansion is proportional to the size of the expanding object. For linear expansion, $\Delta L = \alpha L \Delta T$, where α is the **coefficient of linear expansion**. Alternatively, we can write this as $\Delta L/L = \alpha \Delta T$, so the fractional change in the length of an object is proportional to the change in its temperature. Because $\Delta L/L$ is a dimensionless quantity, we see that α has units of inverse temperature units ($^\circ\mathrm{C}^{-1}$ or, equivalently, K^{-1}).

Similarly, an object will expand in volume according to the formula $\Delta V/V = \beta \Delta T$, where β is the **coefficient of volume expansion**. For small fractional changes in length or volume (actually, most you will ever encounter), we can use the relation $\beta = 3\alpha$, which turns out to be a nifty result from calculus.

In a few days we will learn about ideal gases, which can be characterized by four variables: pressure (P), volume (V), temperature (T), and number of molecules (N). These quantities are related by the **ideal gas law**, $PV = Nk_BT$, where k_B is a fundamental physical constant known at **Boltzmann's constant**.

Let's write $V = \left(\frac{Nk_B}{P}\right)T$. For a fixed amount gas held at constant pressure, $\Delta V = \left(\frac{Nk_B}{P}\right)\Delta T$, so $\frac{\Delta V}{V} = \frac{\Delta T}{T}$. This means that for an ideal gas $\beta = 1/T$. Of course this result is dimensionally consistent with our previous result — the units are those of inverse temperature. I'll leave you with one final question to consider: in the equation $\beta = 1/T$, which temperature scale(s) (Celsius, Fahrenheit, Kelvin, or Rankine) can T be in? Try to justify your answer for each case.