

# Work and Heat

## Introduction

In the previous lecture we discussed how the temperature of a system is related to its internal (or thermal) energy. We also discussed how we could change this energy, and therefore the temperature, by adding heat. In that discussion we restricted ourselves to a case where the volume of the system is constant. The reason for this is because we did not want any energy exchanged with the surroundings as work is done on the system or by the system on the surroundings.

Today we will consider work processes. Like heat, work is another way to add energy to a system. If you squeeze a gas you add energy to it through work. If a gas expands, it loses energy by doing work on its surroundings.

## Thermodynamic Work

In discussing thermodynamic work, an ideal gas is the easiest system to consider. While solids will expand or contract as their temperature changes, these changes in volume are small and usually ignorable. Gases, on the other hand, are very easy to compress or to expand to several times their initial volume. For this reason, work is a major issue for any changes to a gas.

We will consider a gas in a container whose volume is fixed by a moving piston of area  $A$ . When the piston is fixed in place, there will be a certain gas pressure, and therefore a certain force on the piston,  $F = PA$ . If this force causes the piston to move a displacement  $\Delta x$ , the force of the gas on the piston will do an amount of work  $W = F\Delta x$ . When the gas expands,  $F$  and  $\Delta x$  will be in the same direction so the gas will do positive work in moving the piston (and ultimately, on the surroundings). If the gas contracts,  $F$  and  $\Delta x$  will be in opposite directions and the gas will do negative work on the surroundings, i.e. work will be done *on the gas* by the surroundings.

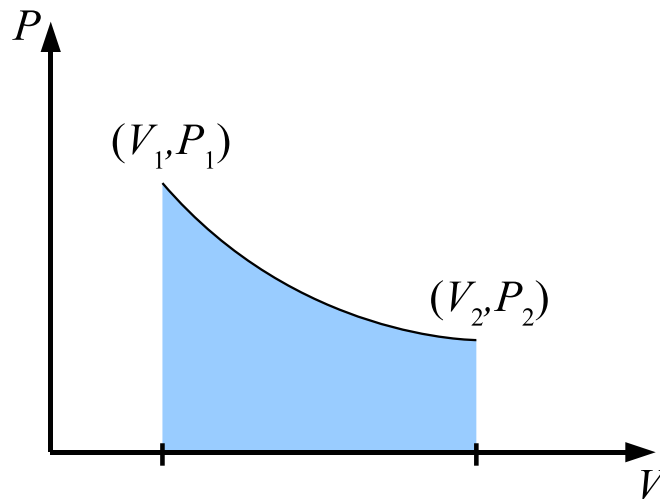
If the piston moves, that means that the volume of the gas will change. If the piston moves displacement  $\Delta x$ , that means the volume of the gas will increase by an amount  $\Delta V = A\Delta x$ . Therefore, the work done by a gas on its surroundings is  $W = (PA)\Delta x = P\Delta V$ , so long as  $P$  is constant.

Unfortunately, the minute the volume of a gas starts changing, it's a good bet its pressure will change, too. Therefore, we need to be a little more mathematically rigorous in defining the work done by an ideal gas. For an infinitesimal change in volume,  $dV$ , the gas will do an infinitesimal amount of work,  $dW = P dV$ . Then, if the volume of a gas changes from  $V_1$  to  $V_2$ , the work done by the gas on its surroundings is

$$W = \int_{V_1}^{V_2} P dV ,$$

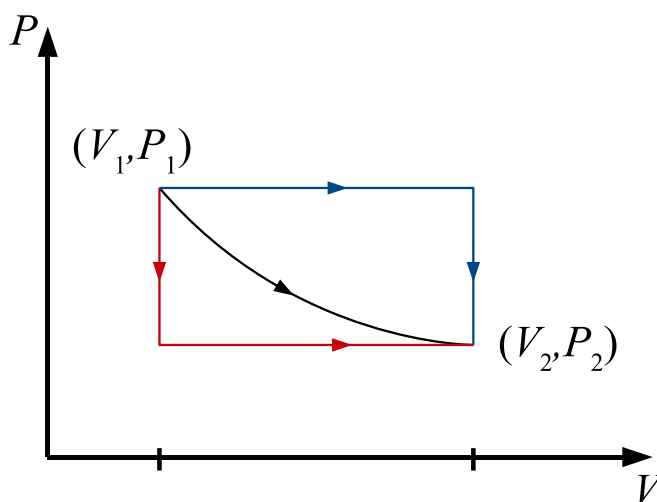
where  $P$  will likely be some function of  $V$ .

It is common to draw a gas process on a diagram called a  $PV$  diagram, in which pressure ( $P$ ) is plotted on the vertical axis, and volume ( $V$ ) is plotted on the horizontal axis. The diagram below shows a process that takes a gas from an initial pressure  $P_1$  and volume  $V_1$  to a final pressure  $P_2$  and volume  $V_2$ . In this case, the pressure drops as the gas expands along the path represented by the black curve.



According to our integral definition of work, the shaded region under the curve connecting  $(V_1, P_1)$  to  $(V_2, P_2)$  is the work done by the gas during the process.

The diagram below shows three different processes that take the gas from  $(V_1, P_1)$  to  $(V_2, P_2)$ . The work done by the gas on its surroundings is different in each case, with most work done in the process represented by the blue path, and the least work done in process represented by the red path. In the blue and red processes above, all of the work is done in the horizontal sections of the processes. No work is done in the vertical parts. In the black process, some work is done along every part of the path.



## The First Law of Thermodynamics

Let's use the symbol  $U$  to represent the total internal (thermal) energy of a system. (Yes, this is the same symbol we use for potential energy. Unfortunately, there are a finite number of letters in the alphabet.) We now know of two ways to change the thermal energy of the system: we can add heat to it, or we can do work on it. The symbol  $Q$  represents the *heat added* to the system.  $Q$  is positive when the heat is added (increasing the thermal energy), and  $Q$  is negative when the heat is removed (decreasing the thermal energy). As defined above, the symbol  $W$  represents the work done *by the system* on its surroundings.  $W$  is positive when the system does work on the surroundings (decreasing the thermal energy) and  $W$  is negative when the surroundings do work on the system (increasing the thermal energy). Therefore, the law of conservation of energy can be stated mathematically as

$$\Delta U = Q - W \quad \longrightarrow \quad dU = dQ - dW = dQ - P dV .$$

These mathematical expressions are examples of the **first law of thermodynamics**. The first is an expression for finite changes to a system, while the second is for an infinitesimal change. (Note the distinction between “ $d$ ” and “ $d$ ” in the above equations:  $dU$  and  $dV$  are infinitesimal

*changes* in energy and volume, while  $dQ$  and  $dW$  are infinitesimal *amounts* of heat and work. The distinction is not that important for us, but it's worth noting.)

Does the sign convention seem strange to you? It should. Why would  $Q$  be positive when we add energy to the system through heat, while  $W$  is positive when we take energy away from the system through work? The reason is historical, having to do with the origin of thermodynamics as the search for better, more efficient heat engines (steam, internal combustion, etc.) As we will see next week, a heat engine often uses a gas as the working system. It takes heat in, and is used to do work on its surroundings. That is the origin of the sign convention above. Many books use a different sign convention. I'll teach you about them when you take my *Thermal Physics* class. (You *are* planning to take it, right?)

As long as the number of molecules in a gas is fixed, then for each point in a  $PV$  diagram, the values  $P$  and  $V$  determine the temperature,  $T = (PV)/(Nk_B)$ , which in turn determines the total thermal energy of the gas. For a gas with  $f$  degrees of freedom per molecule is  $U = (f/2)Nk_B T = (f/2)PV$ . We say that each point on a  $PV$  diagram represents a **state** of the gas, and a change in the system that takes it from one point to another in a  $PV$  diagram corresponds to a **change in state**. In many cases, you can tell just by looking at a path on a  $PV$  diagram whether the thermal energy increases or decreases as the state of the gas changes. For example, a change of state that follows a path to the right (constant  $P$ , increasing  $V$ ), indicates an increase in thermal energy. As we will see next time, it is not always easy to tell by sight whether a change in state corresponds to a change in energy and temperature.

Now let's go back and consider the second  $PV$  diagram above, that shows three different paths the gas could take as its state changes from  $(V_1, P_1)$  to  $(V_2, P_2)$ . For all three paths,  $\Delta U$  and therefore  $\Delta T$  will be equal—these are *path-independent* quantities. However, the work done by the gas is different for each path. Work is a *path-dependent* quantity. This means that heat is also a path-dependent quantity.

Even if we can't tell how the total energy changes in a gas as it changes state in a  $PV$  diagram, there are many things we can tell. As we saw above, any process that corresponds to an increase in volume (moving, at least in part, to the right in the  $PV$  diagram) represents positive work done by the gas. Any process that represents a decrease in volume (moving to the left in the  $PV$  diagram) represents negative work done by the gas (work is done on the gas). The greater the area under the path representing the change of state, the greater the work done by (or on) the gas. It is easy to compare the work done by a gas along the different paths, just by sight.

We can also tell something about the heat added to the gas, as well. Re-write the first law of thermodynamics as  $Q = \Delta U + W$ . In all three paths in the above diagram,  $\Delta U$  will be the same. Therefore, the heat added to the gas will be greater for the path with the greatest work done by the gas. We may not be able to tell by sight how much heat is absorbed by the gas in the blue process, but we know that it absorbs more heat than when the gas follows the black path, which in turn absorbs more heat than when the gas follows the red path.