Electric and Gravitational Fields

After last class, you may be wondering how two charges or two masses can exert forces on each other when they are separated by great distances. How does the charge (or mass) know where the other charge (or mass) is? The answer is easy to come by when we introduce the concept of the **field**. The field concept is useful for any "action at a distance" forces.

Consider an isolated charge, Q at some point in space. Now let's place a second *test* charge, q, at distant point away from Q. We can observe q and measure an electric force on it. If we had put q somewhere else, we would have measured a different force on it (stronger or weaker, or possibly pointing in a different direction). If we had put a larger test charge, say $2q$, we would have measured that there is twice the force on $2q$ as there is on q. Coulomb's law says that it is Q that exerts all of these forces on our test charges, but how does Q "know" to exert different forces when our test charges are at different locations or have different charge values?

Rather than trying to imagine that Q instantaneously changes the force it exerts based on the value and location of the test charge, imagine instead that Q creates a new physical entity, known as the **electric field**. This field, located everywhere in space, is like a set of instructions that tell the test charge how to react electrically when placed at a given point. When we put q at some point, it does not react directly to the distant charge Q (located somewhere "over there"). Rather, it reacts to the field created by Q, that is located at the point where our test charge is placed ("right here").

The same applies to gravity. The sun does not exert a force on Earth directly. Rather, the sun creates a gravitational field that fills space. At any given point in its orbit, Earth reacts to the sun's gravitational field at that point.

Electric Field

Recall that the magnitude of the electric force between charges Q and q_1, separated by a distance r, is $F_e = k_e|Qq_1|/r^2$. Let's focus on the force on charge q_1, and for simplicity say that q_1 is located "here", while Q is located "there." If we were to take q_1 away and replace it with q_2 at the same location ("here"), the force on the new charge will be $F_e = k_e|Qq_2|/r^2$. That means the force on a particle "here" is proportional to the charge of the particle that is located

"here." Let's write this mathematically as $F_e(\text{here}) = q_{\text{here}}\, E$. The quantity E depends only on the charge "there" (Q) and the displacement from "there" to "here." From Coulomb's law, therefore, we see that the electric field created by Q has a magnitude $E = k_e |Q|/r^2$ at all points a distance r from Q. Note that E should have SI units of newtons per coulomb, N/C.

So far we have only considered the magnitude of the electric field. What about its direction? Let's assume that Q is a positive charge. If we put a positive test charge $+q$ at some point, it will experience a force directed away from Q. If, instead we put a negative test charge $-q$ at the same point, it will experience a force directed toward Q. To account for this, we say that the electric field vector, $\vec{\mathbf{E}}$ "points away" from Q. Then the force on $+q$ points in the same direction as $\vec{\mathbf{E}}$, while the force on $-q$ points in the opposite direction of $\vec{\mathbf{E}}$. The correct vector expression relating field to force, therefore, is $\vec{\mathbf{F}} = q_{\text{here}}\, \vec{\mathbf{E}}$. The field $\vec{\mathbf{E}}$ points in the direction of the force that a positive test charge would feel here.

What if Q were negative? The force on a positive test charge $+q$ would point toward Q, while the force on a negative test charge $-q$ will point away from Q. Therefore, we can see that the electric field created by a negative charge points toward that charge.

Gravitational Field

Using Newton's law of gravitation, we see that the gravitational field behaves much like the electric field. If we have a point mass M, then the force on a test mass m_1 a distance r away from M will experience a force of magnitude $F_g = GMm_1/r^2$. A test mass m_2 will experience a force of magnitude $F_g = GMm_2/r^2$. Therefore, as with the electric field, we can write the force on a test mass m is $F_g = m\, g$, where g is the magnitude of the gravitational field at the location of the test mass. Note that g should have SI units of newtons per kilogram, N/kg.

Now you know why we called g the "gravitational field strength" last term. Near Earth's surface, the gravitational field strength is $g = 9.8\,\text{N/kg}$, while the gravitational acceleration is $a_g = 9.8\,\text{m/s}^2$. (Why these two values are equal in value and dimension is a deep question of physics and gives rise to the so-called "equivalence principle" Einstein used to formulate his General Theory of Relativity. For our purposes, we will just distinguish between "gravitational field strength" and "gravitational acceleration" and note that most textbook authors do not.)

Field Lines

Electric and gravitational fields are vector quantities; they add together as vectors. If, rather than a point charge, you have a complex distribution of charge, you must compute the field at each point by breaking that complex source up into tiny bits of charge, computing the field at the desired point due to each tiny bit of charge, then add up all the fields vectorially. The same is true for gravitational fields. The process can be quite cumbersome mathematically.

However, for many qualitative purposes, it is easier to imagine roughly what the field looks like at each point. Using field vectors can be rather complicated. For that reason, physicists often represent fields by sets of "field lines" that fill space around charges. Here are a few basic rules for field lines:

- Field lines only begin or end on charges, or extend to/from infinitely far away. They never begin or end on random points in empty space where there is no charge.
- Field lines always point away from positive charges, and always point toward negative charges.
- The number of field lines one draws can be arbitrarily large. However, you should make them proportional to the charges in your problem. If you draw four field lines extending outward from a charge q, then in the same diagram you should draw eight field lines extending outward from a charge $2q$.
- The electric field vector at any point is tangent to the field line at that point.
- The strength of the electric field (magnitude of the field vector) at a point depends on the density of field lines in that region. More closely spaced lines in a region indicate a greater electric field in that region.

The rules above hold for gravitational fields, with one exception: gravitational field lines always point *toward* masses, never away.