Capacitance

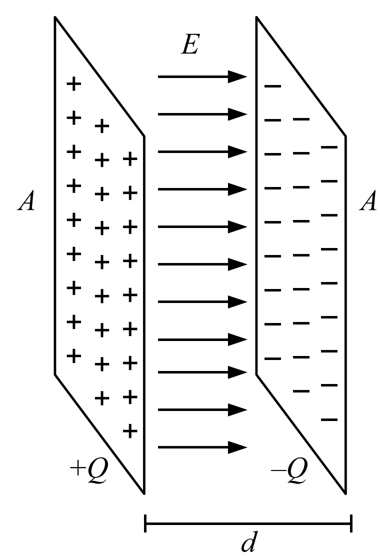
Consider two equal and opposite point charges, $\pm Q$, separated by a distance r. The potential energy of the interaction between them is

$$U = -\frac{k_e Q^2}{r} \ .$$

That means that the interaction stores a certain amount of energy. This stored energy is less than if they were infinitely separated because you would have to do work, adding energy to the system, to separate them infinitely far apart. However, the energy is greater than if the charges were closer together because the field will do work (you do negative work) as the charges attract each other ever closer.

This interaction energy is actually stored *in the electric field* itself. In the next two classes we will learn more about this energy, first by investigating the concept of capacitance.

Capacitance and Capacitors



The figure on the right shows two large, closely spaced parallel charged conducting plates, which carry equal and opposite charges.

As long as the side length of the plates is much larger than the separation between them, the equipotential surfaces will be parallel to the plates and equally spaced, making the electric field uniform between the plates. In reality, as you saw in lab, the equipotentials and field lines will deviate from uniformity near the edges, but as long as the plates are close enough compared to their size, this effect is small. (The picture here has a much larger separation than we require so that I can show the charges and the uniform field between the plates.)

The configuration above is an example of a **capacitor**, an electronic device that stores charge Q when there is a potential difference ΔV between its plates.

For simplicity, you can imagine a field line starting on each positive charge on the left plate and ending on each negative charge on the right plate. Now imagine we double the charge on each plate. The electric field lines will still be uniform, but there will be twice as many of them as before—the field strength will have doubled. The potential difference between the plates will also double.

This result can be generalized to any geometry, as long as the geometry is fixed and we only change the charges. This leads to a very important result: *For any capacitor of fixed geometry, ΔV and Q are directly proportional. Therefore, their ratio is a constant that characterizes all the construction details.* We call this ratio **capacitance**: $C = Q/\Delta V$. Capacitance tells us how much charge can be stored on the plates for a given potential difference between the plates. It depends entirely on the geometry of the plates, and possibly the medium they are placed in.

Note on symbols and units: V is the symbol for electric potential, while V is the symbol for "volts," the SI unit of potential. C is the symbol for capacitance, while C is the symbol for "coulombs," the SI unit of *charge*, not capacitance! The SI unit of capacitance is the "Farad": $1\,\mathrm{F} = 1\,\mathrm{C/V}$. In general, italic letters refer to physical quantities and upright letters refer to units.

Parallel Plate Capacitors

The geometry shown above is an example of a **parallel plate capacitor**. Again, parallel plate capacitors are characterized by the field being uniform between the plates. Through a bit of fun math one can show that if the plates of a parallel plate capacitor have area A, then the field between them is

$$E = \frac{4\pi k_e Q}{A} = \frac{Q}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0}\,,$$

where $\epsilon_0 = 1/4\pi k_e$ is a constant called the "permittivity" of the vacuum and $\sigma = Q/A$ is the surface charge density on the plates. Because the field is uniform, we can write

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}\,,$$

where d is the separation between the plates. The capacitance of a parallel plate capacitor in a vacuum (a good approximation for air), therefore, is

$$C = \frac{\epsilon_0 A}{d}\,,$$

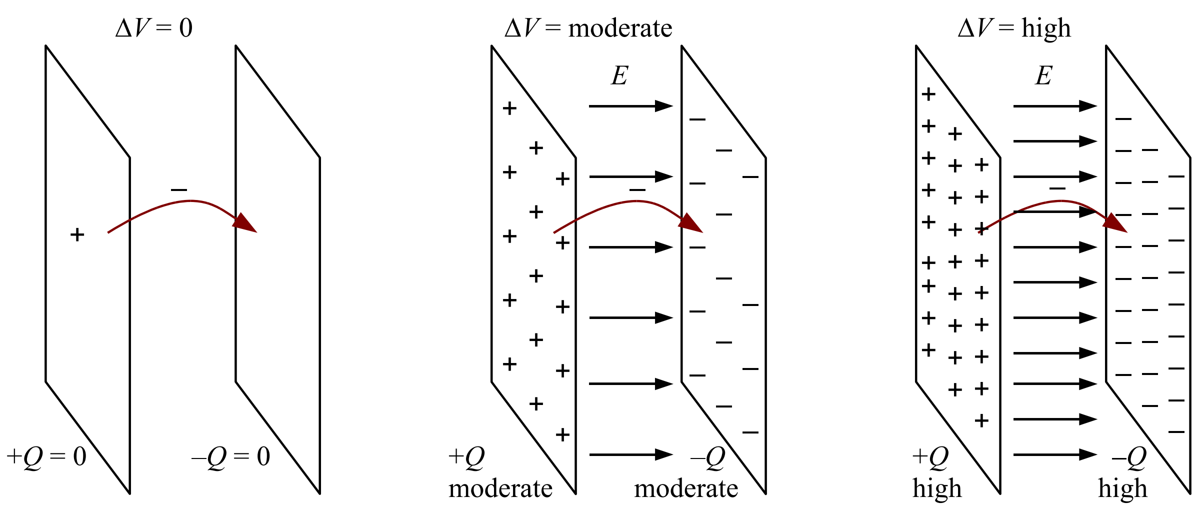
a quantity that depends entirely on the geometry of the charges (area A and separation d) and the medium (ϵ_0 is a property of the vacuum of space). To make a stronger capacitor, you can either make the plates larger or more closely spaced.

Energy Stored in a Capacitor

Because there's a potential difference between the charged plates of a capacitor, you will not be surprised to learn that there is an electric potential energy associated with a charged capacitor. It would be nice to say that the potential energy is simply the product of the charge and the potential difference, as it was for point charges. However, we are not so lucky as to have this—the charged plates of a capacitor are not as simple as a test charge.

The easiest way to determine the energy stored in a capacitor is to determine how much energy is required to charge it. Imagine that we initially have two neutral parallel plates of area A, separated by a distance d. There is no charge on the plates, so there is no potential difference and no energy stored in the capacitor.

How do we charge the plates? Recall that we want one plate to be charged $+Q$ and the other to be charged $-Q$. One way to make this happen is to take a small amount of negative charge (say one electron) from one plate and move it to the other. Initially it takes very little work to do this because the electron will only be slightly attracted to the left plate, which now has a slight net positive charge. However, if we continue moving one electron at a time, it takes more and more work to move successive electrons, which are repelled by the negative plate and attracted to the positive plate as the potential difference, and hence the field, increase. (See figure below.)



For each small charge dq moved near the beginning of charging, the change in potential energy is $\Delta U = \Delta V_{\text{initial}}\, dq \approx 0$. For each small charge moved near the end of charging, the change in potential energy is essentially $\Delta U = \Delta V_{\text{final}}\, dq$. On average, the increase in potential energy moving a charge dq is approximately $\Delta U = (1/2)\Delta V_{\text{final}}\, dq$. Therefore, the total potential energy of the charged capacitor is

$$U = \frac{1}{2}\Delta V_{\text{final}}\, Q = \frac{Q^2}{2C}\ .$$