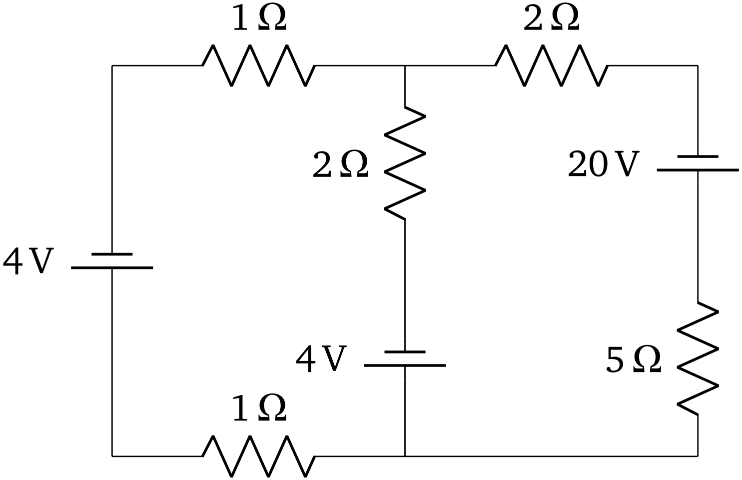
## Kirchhoff’s Rules



The circuit below consists of two identical resistors and a pair of batteries, one 12 volt, the other 6 volt, in opposition to each other, as shown. The current through the resistors is

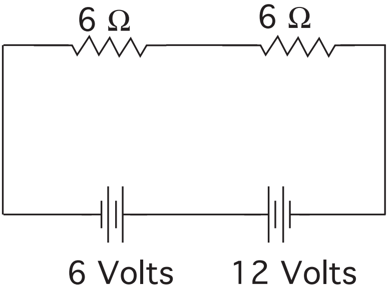
A. 3 A

B. 1.5 A

C. 1 A

D. 0 A

E. 0.5 A



**ANS: E**—The current is $I = 0.5\,\mathrm{A}$

This is a simple application of Kirchhoff's loop rule. All circuit elements are wired in series with each other, so all have *the same* current, $I$, passing through them. I will assume that the current flows clockwise through the circuit.

Start to the right of the 12 V battery and trace clockwise around the circuit. The sum of the potential changes around the loop is

$$+12\,\mathrm{V} - 6\,\mathrm{V} - (6\,\Omega)\,I - (6\,\Omega)\,I = 0 \quad\Longrightarrow\quad 6\,\mathrm{V} = (12\,\Omega)\,I\,,$$

giving $I = 0.5\,\mathrm{A}$.

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B. 1.5A

C. 1A

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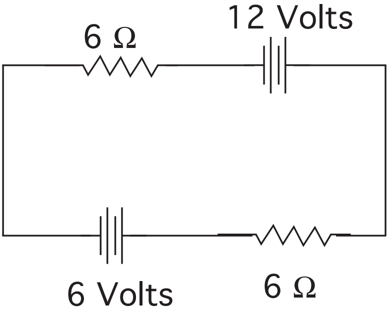

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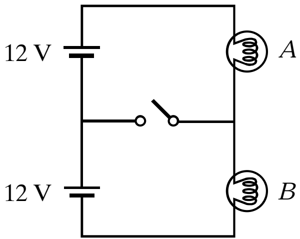
giving $I = 0.5\,\mathrm{A}$.

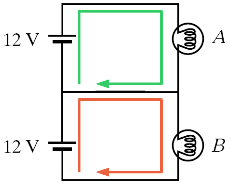
The light bulbs in the circuit are identical. When the switch is closed,

A. both go out.
B. the intensity of light bulb $A$ increases.
C. the intensity of light bulb $A$ decreases.
D. the intensity of light bulb $B$ increases.
E. the intensity of light bulb $B$ decreases.
F. some combination of A–E occurs.
G. nothing changes.



**ANS: G**—Nothing changes when the switch is closed.

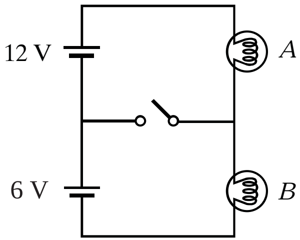
This is a good example of when to use the loop rule. When the switch is open, there is only one loop. The total potential difference across the two bulbs is 24 V. The current is *the same* through the bulbs, which have equal resistances. Therefore, the potential differences across the bulbs will be equal. Because these potential differences must add to 24 V total, we know that the potential difference across each bulb will be 12 V.




Now consider the case when the switch is closed (diagram above). Along the green loop our potential rises by 12 V, then drops by $\Delta V_A$ (the potential difference across *A*) before returning to the starting point. Therefore, the potential difference across *A* is 12 V. This is equal to the potential difference before the switch was closed, so the brightness of bulb *A* will not change. An analysis of the orange loop leads to the same conclusion for bulb *B*. Interestingly, this also means that the currents through *A* and *B* will be equal (the same?) so there must be no current flowing through the closed switch.

**Exercise:** Try to answer this question in the case where the batteries are still identical, but bulb *A* has more resistance than bulb *B*.

The light bulbs in the circuit are identical. When the switch is closed,

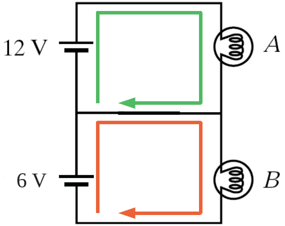



A. both go out.
B. the intensity of light bulb $A$ increases.
C. the intensity of light bulb $A$ decreases.
D. the intensity of light bulb $B$ increases.
E. the intensity of light bulb $B$ decreases.
F. some combination of A–E occurs.
G. nothing changes.

**ANS: F**—The intensity of bulb *A* increases, while the intensity of bulb *B* decreases.

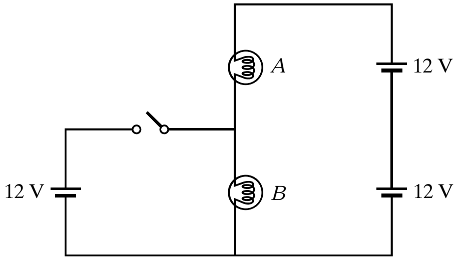
Following the analysis in the previous question, when the switch is open the two bulbs have a total potential difference of 18 V across them, so each bulb has a potential difference 9 V across it. The bulbs will be equally bright.

Now consider the case when the switch is closed, as shown in the diagram at the below. Analyzing the green loop, we see that the potential difference across bulb *A* rises to 12 V. Therefore, bulb *A* gets brighter. Analyzing the orange loop, we see that the potential difference across *B* is 6 V, so bulb *B* gets dimmer.




Because the bulbs have equal resistances, the current through bulb *A* will be twice as large as the current through bulb *B* when the switch is closed. Therefore, unlike the previous question, we see that there will be a current through the horizontal branch. The large current through *A* is split equally. Half will go down through bulb *B*, while the other half will move right-to-left through the closed switch.

The light bulbs in the circuit are identical. When the switch is closed,




A. both go out.

B. the intensity of light bulb $A$ increases.

C. the intensity of light bulb $A$ decreases.

D. the intensity of light bulb $B$ increases.

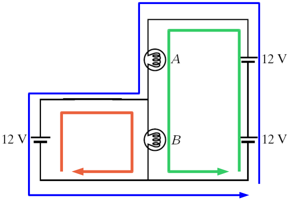
E. the intensity of light bulb $B$ decreases.

F. some combination of 1–5 occurs.

G. nothing changes.

**ANS: G**—Nothing changes.

With the switch open, there is only one loop. The left-hand battery does not contribute any current. The currents through A and B are the same, so the potential differences across $A$ and $B$ will be equal: $\Delta V_A = \Delta V_B = 12\,\mathrm{V}$.




Now consider the system with the switch closed (diagram above). Following the red loop, the potential difference rises 12 V passing the battery, so it must drop by 12 V passing through $B$ and returning to the starting point. Therefore, the potential difference across $B$ remains 12 V, so the brightness of $B$ will not change. Following the green loop, and using $\Delta V_B = 12\,\mathrm{V}$, we conclude that $\Delta V_A = 12\,\mathrm{V}$. Thus the brightness of the bulbs will not change. (**Exercise:** try the blue loop in this analysis.)

Because bulbs $A$ and $B$ are identical and the potential differences across them are equal, the currents through them will be equal. According to the junction rule for the point between $A$ and $B$, equal amounts of current enter the junction from $A$ and leave the junction for $B$. Therefore, even with the switch closed, the left-hand battery contributes no current to bulb $B$. (This may seem surprising to you. Go back over the analysis to convince yourself of this.)

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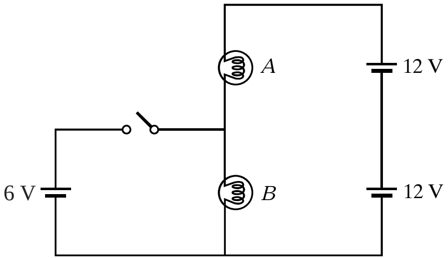



A. Bulb $A$ is unaffected

B. The current through light bulb $B$ increases

C. The current through light bulb $B$ decreases

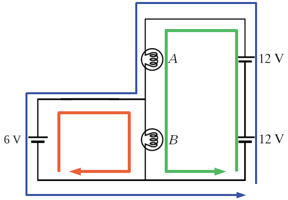
D. Bulb $B$ has 2 currents going through it

E. The potential drops across the light bulbs are equal to each other

F. More than one of the above is true

**ANS:** **C**—The current through bulb $B$ decreases.

Prior to the switch closing, the potential difference across the series combination of bulbs $A$ and $B$ is 24 V. The same current flows through the two identical bulbs, so they will have equal potential differences $\Delta V_A = \Delta V_B = 12\,\text{V}$.




Now consider the situation with the switch closed (picture above). According to the blue path, the potential difference across bulb $A$ is 18 V, so bulb $A$ will get brighter after the switch is closed. According to the red path, the potential difference across bulb $B$ is now 6 V, meaning that bulb $B$ will get dimmer after the switch is closed.

This finding is also verified by the green path. The potential difference across the series combination of bulbs $A$ and $B$ is still 24 V. After the switch is closed, $\Delta V_A + \Delta V_B = 18\,\text{V} + 6\,\text{V} = 12\,\text{V}$, as we should expect.

This also leads us to conclude that the current through $B$ will be less than the current through $A$. Apparently, two-thirds of the current through $A$ will branch off and pass through the 6 V battery!

## Warmup Question

What would happen if you connected a pair of unequal batteries in parallel, say, a 12 Volt and a 9 Volt battery? Explain your reasoning.

**ANS:** This is a potentially confusing question. What does it mean for the batteries to be in parallel? If we connect the positive side of one to the negative side of the other, and vise versa, then we can reasonably say that the batteries will be in series. Then the EMFs of the batteries will add, as will the internal resistances, and we will get a high current through the wires.

Wiring the batteries in parallel, therefore, means connecting the two positive terminals together and the two negative terminals together. This results in a structure very much like the second question above, with the two batteries seen as EMFs and the two resistors seen as (much smaller) internal resistances. Then the current will flow out of the positive terminal of the battery with the larger EMF, and into the positive terminal of the battery with the smaller EMF. If the battery with the smaller EMF is rechargeable, it will be recharged.

## Warmup Question

Analogy: Charge is for a capacitor as what is for a resistor?

A. energy
B. power
C. current
D. potential

**ANS: C**—Charge is for a capacitor as **current** is for a resistor.

Capacitors in series all have the same total charge on them. Resistors in series have the same current through them. The energy stored in a capacitor is related to the product of charge and potential difference. The energy dissipated by a resistor is related to the product of current and potential difference.