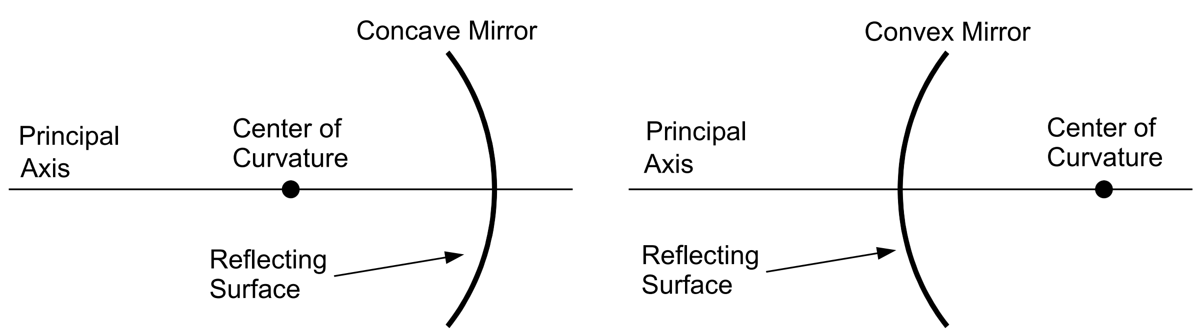
Spherical Mirrors

While it is easy to follow the rays reflected by flat mirrors, the more interesting cases are when the mirrors. Following the rays reflected from a curved surface can be complicated, but remarkably good approximations can be developed for **spherical mirrors**.

Center of Curvature and Focal Point

Spherical mirrors are so-named because they can be seen as sections cut out of a much larger spherical surface. A spherical surface is defined as the set of all points that are the same distance from a given point, the **center of curvature**. The **radius of curvature** is the distance from the center of curvature to any point on the surface. For our purposes we define the **principal axis** as the line that goes through the center of curvature and the center of the mirror.

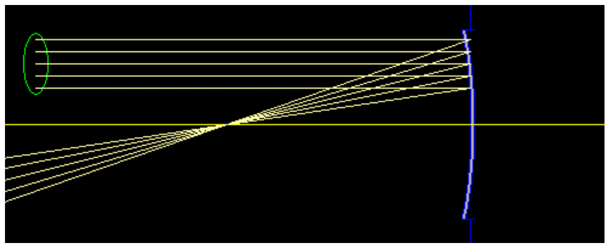
The diagram below shows two examples of spherical mirrors, **concave** (or **converging**), where the reflecting surface faces toward the center of curvature, and **convex** (or **diverging**), where the reflecting surface faces away from the center of curvature.



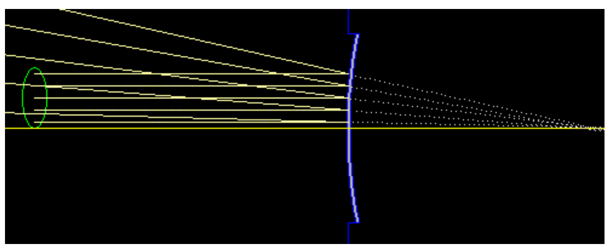
Now let's consider what happens when **paraxial** light rays hit spherical mirrors. Paraxial rays are those that are initially parallel to the principal axis. Spherical mirrors are easy to study

because they have a **focal point** toward which, or away from which, paraxial rays are reflected. Consider the diagrams below.

Converging mirrors reflect paraxial rays toward the focal point.



Diverging mirrors, on the other hand, reflect paraxial rays away from the focal point.



The dotted lines in the image above show where the diverging rays appear to be coming from. These rays are behind the mirror, so there are no actual rays there.

Notice the the focal point of a spherical mirror is on the same side of the mirror as the center of curvature. In fact, the focal point is *half way* between the mirror and the center of curvature.

Forming Images and Ray Tracing

If an object is a point source of light, or a collection of paraxial rays, the image will be focused at a single point. For extended objects, on the other hand, the image will also be extended. Therefore, to find the image we would have to treat each point on the object as a source and find the image point for this source. The collection of all image points is the image formed by the mirror. (Consider looking at yourself in the mirror. Your the image of your left eye is in one location, your right thumb tip in another, etc.)

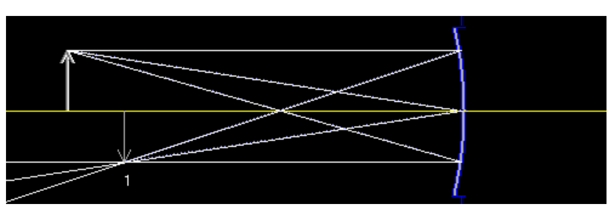
To find where the image of a point source is located, need to trace multiple rays from the source and find where they cross (focus). There are an infinite number of rays that will all focus at the same point, but we only need a few. We could use two rays, but if there is a slight tracing error we would have no way to know it because two rays cross at a single point. It's better to use three. Luckily, there are three **principal rays** that are easy to trace:

- A paraxial ray through the source point. This ray will reflect toward or away from the focal point.
- A ray through the source point that approaches or passes through the focal point. The reflected ray will be paraxial.
- A ray through the source point that hits the mirror at the principal axis (the **vertex**). The ray will reflect away from the principal axis at the same angle.

We are interested in the simple case where the object is located at a specific distance from the mirror. For that reason, we choose to make our prototype object a "stick" that may or may not cross the principal axis. As we will see, sometimes the image is rightside-up, and other times upside-down. Therefore, it's even more useful to make our object an arrow.

The cool thing about an arrow object is that we really only need to trace the principal rays of a single point. If we put the tail of the arrow on the principal axis, we know the image of the tail will be located somewhere on the principal axis. That means we only need to find the image of the head of the arrow. This locates the image location (distance from the mirror). By convention we always place the object arrow upright and to the left of the mirror.

Let's take a look at some examples. First, consider the image formed by a converging mirror when the object is farther from the mirror than the focal point. The object is the upright arrow, while the image is the inverted arrow denoted by the "1".

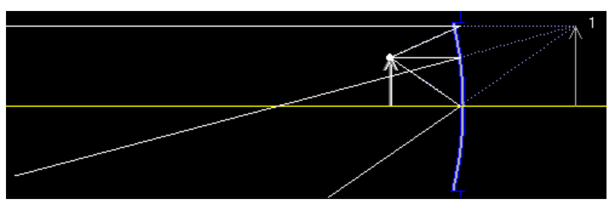


There are several features you should notice about the image.

- It is on the same side of the mirror as the object. The rays focus to a point. If you were to put a piece of film at that point, you could capture the image. That is why we call such images **real**. If the rays appeared to diverge from a point behind the mirror, we would call the image **virtual**.
- The image is **reduced**, or smaller than the object. If the image were larger than the object, we would say the image was **magnified**.
- The image is **inverted**, or upside-down compared to the object. An **upright** image is oriented in the same direction as the object.

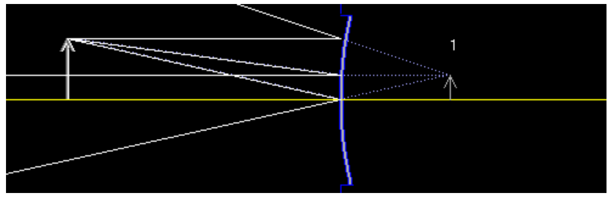
In summary, the image produced by the mirror in the above example is *real*, *reduced*, and *inverted*. In general, any object outside the focal point of a converging mirror will produce a real, inverted image. The image will be magnified if the object is between the focal point and center of curvature, and will be reduced if the object is beyond the center of curvature. If the object were at the center of curvature, the image would also be located at the center of curvature.

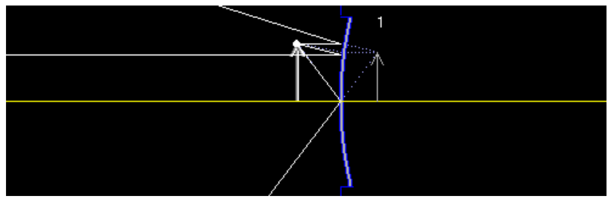
Now consider a converging mirror with the object located between the focal point and the mirror.



The reflected rays do not cross. Instead, they all diverge from a point behind the mirror. The resulting image is *virtual*, *magnified*, and *upright*. In general, any object inside the focal point of a converging mirror will produce a virtual, upright, magnified image.

Finally, consider images produced by a diverging mirror. The first is for an object outside the focal point, while the second is for an object inside the focal point.





In both cases, the images are *virtual*, *upright*, and *reduced*. This is true for all images formed by a diverging mirror.

Finding Images Algebraically

While it is often useful to trace principal rays to get qualitative information about the image produced by a mirror, we can find the image more precisely through simple formulas. Let R be the radius of curvature of the mirror. Then the **focal length** of the mirror, the distance between the focal point and the mirror vertex, is $f = R/2$. For a converging mirror, $R > 0$ and $f > 0$. For a diverging mirror, $R < 0$ and $f < 0$.

The **object distance**, s, is the distance between the object and mirror vertex. The **image distance**, s', is the distance between the image and the mirror vertex. The object distance is typically positive, $s > 0$, because the object is typically on the side of the from which the rays are coming. (It is possible to have $s < 0$ in certain cases. If you're good, maybe I'll tell you about that in class.)

The signs for image distances depend on where the image is located. A real image, located on the side of the mirror to which the rays are reflected, has a positive image distance, $s' > 0$. A virtual image is defined to have a negative image distance, $s' < 0$.

We can relate the object and image distances with the focal length of the mirror by the simple equation

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \Longrightarrow \quad s' = \frac{sf}{s-f} \ .$$

The equation on the right allows us quickly to verify the results we found above with the ray tracing. For a diverging mirror ($f < 0$), the image distance is always negative ($s' < 0$) so the

image is always virtual. For a converging mirror, the image distance is only positive, and hence the image is only real, when $s > f$.

Furthermore, in the limit in which curved mirrors become flat ($f \longrightarrow \pm\infty$) we have $s' \longrightarrow s$.

Magnification is also related to object and image distances and object and image heights. Let h be the height of the object, and let h' be the height of the image. An upright image has $h' > 0$, while an inverted image has $h' < 0$. Then we have

$$M \equiv \frac{h'}{h} = -\frac{s'}{s} \ .$$

Next time we'll see what happens with refraction by thin lenses rather than reflection by spherical mirrors.