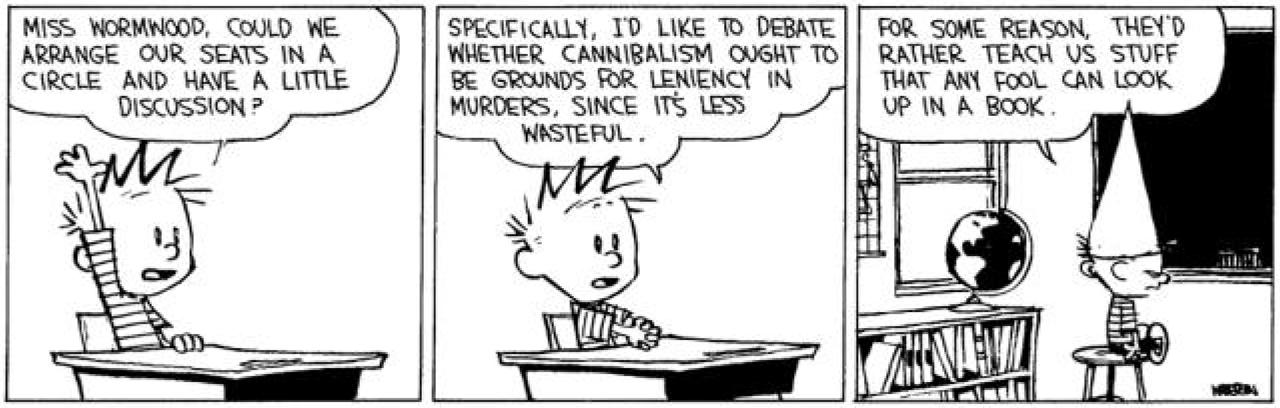


Sources of magnetic fields

We've learned that magnetic fields exert forces on electric charges only if they're moving. Furthermore, they don't do it in anything like a simple manner. The next question to consider is the source of magnetic fields. For contrast, let's review the basic properties of electric fields:

- Electric fields exert forces on electric charges
- The force is directed parallel to the electric field
- Electric charges create electric fields
- Point charges create electric field directed radially toward/away from them

In other words, electric charges both create and respond to electric fields, so everything seems nicely self contained. However, there simply aren't any magnetic charges (a.k.a., monopoles), so we can't expect an analogous situation to hold for magnetic phenomena. In fact, we found that the forces from magnetic fields act perpendicularly to what an electric field would do. It turns out that magnetic fields are created in just about as complicated a fashion. Here's a quick overview:

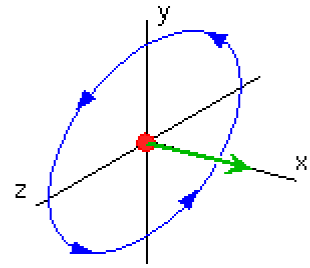
- Magnetic fields exert forces on *moving* electric charges
- The force is directed *perpendicular* to the magnetic field
- *Moving* electric charges create magnetic fields
- Moving point charges create magnetic field directed circularly, *around* the charge and its velocity

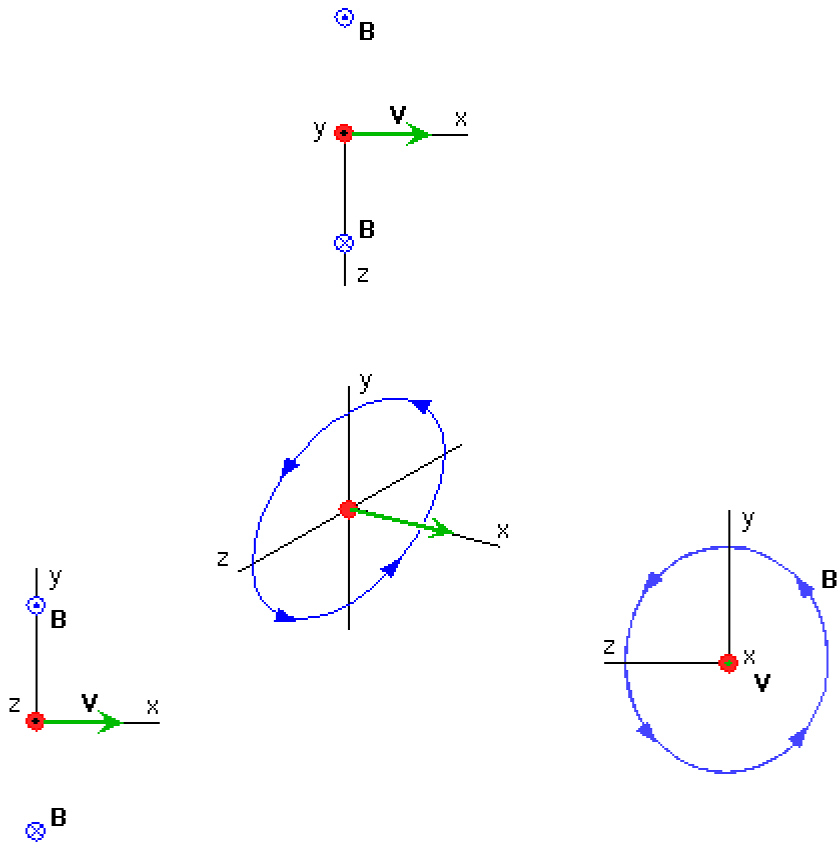
I'll bet electric fields are starting to look downright simple to you. The additional complication arises from the dependence of magnetic phenomena on the velocity vector of the charge and not simply on its magnitude. It should make sense that the magnetic field depends on the distance from its source to where it acts, just like the electric field. What's curious is that the magnetic field isn't simply directly toward or away from the source, as the electric field is. What's different is that the source has a direction associated with it, namely its velocity, and both those vectors must play a role. When you have two vectors as inputs and must get a single vector as your output, the cross product is the only simple option.

Magnetic fields from moving charged particles

Let's consider the third point regarding how moving charges create fields. This figure shows a positively charged particle (red dot) moving with its velocity (green arrow) along the x axis. The magnetic field line shown (blue) is a closed circle lying in the yz plane and centered on x-axis. This is a perspective view, so the field line looks like an oval, but it is really a circle in the y-z plane. The direction of the field around the velocity is determined using your right hand, as follows: point your thumb in the direction of the charge velocity. The field lines curve around the trajectory in the direction that your fingers curl.

These three-dimensional can be difficult to picture, so the picture below shows the same figure in the center but the surrounding frames are projections onto three coordinate planes through the origin. Remember that a vector out of the page is indicated by a circle with a dot in the center (your view of an arrowhead pointed toward you) while a vector into the page is a circle with an "x" across it (your view of the feathers of an arrow pointed away from you). Take some time to study each version and reconcile it with the other versions. You should have a clear impression of the three-dimensional structure before proceeding.





Although only one field line is shown in the figure, there is actually a magnetic field everywhere. Just as the case for electric fields, one sketches a few lines to indicate the general behavior of field elsewhere. In this situation, all magnetic field lines obey the following conditions: They

- lie in two-dimensional planes perpendicular to the x-axis,
- form circles centered on the x-axis, and
- have a right-hand circulation around the axis identical to the one shown.

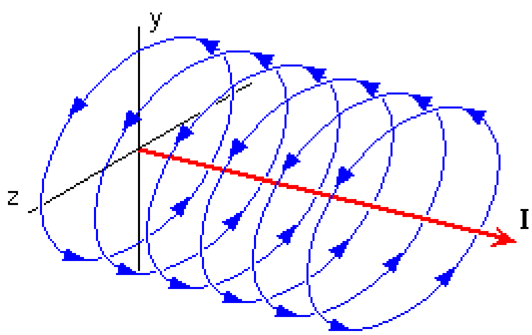
Those rules specify the direction of the field everywhere, now we just need instructions for the field strength, i.e., the magnitude of **B**. Basically, the field strength decreases with distance from a moving point charge, probably as you'd expect. However, the formal expression for the field from a moving point charge is not used very often, so we'll not go into it further. What's more important are expressions for the field produced by a continuous current, which is the next topic.

A final note: What about the magnetic field produced by moving negative charges? Everything is the same, just calculate the field that would be produced by a positive charge moving in that same direction, then reverse the field direction everywhere.

Magnetic fields from currents

A full description of the magnetic field from a moving point charge is somewhat complicated, and the field continually changes as the charge moves along. Fortunately, one rarely deals with that situation. Rather, steady currents involve the continuous, ordered motion of charged particles, so the magnetic fields they produce are independent of time. This is the subject of <u>magnetostatics</u>, i.e., steady currents and magnetic fields that don't change in time. Note that a current implies moving charges, so magnetostatics doesn't require stationary charges. However, as a moving charge changes position, another charge must immediately take its place.

The simplest example of a magnetic field source is a straight line current of infinite length. All you do is combine the individual fields from all the moving charges along the entire current. This implies that the magnitude of the magnetic field is proportional to the current strength, i.e., B ,àù I, because the current depends upon how many charges are flowing. All the field lines are loops centered on the current. Here are some examples of field lines (blue) around a segment of line current (red):



As for a single moving charge, the field curls around the current in a right handed sense: Just grasp the current with your right thumb along the current and your fingers curl in the direction of the field.

For this situation the field strength decreases as the inverse distance from the line current rather than as the square of the inverse distance, which was the case for a point charge. The reason is either simple or hard, depending upon how you look at it[*], but it's a good rule to remember. Anyway, the shortest distance from some position to a line current is denoted "a" (reserving "r" for the distance to the origin), and putting this together with the dependence of B current gives B ,àù I/a. Physically, there's nothing else in the situation that the field can depend upon, so the only thing left to consider is a proportionality constant. The full expression for the field from a line current is B = (Œº$_{o}$/2œÄ) I/a, where Œº$_{o}$ ("mu-naught") is simply another of those universal constants, called the permeability of free space, with value 4œÄ * 10^{7} T*m/A.

What if the current isn't infinitely long? Just as in the electric case, the magnetic field from a current segment of finite length is basically a compromise between the field produced by a moving point charge and that produced by an infinite line current. That is, when you're relative close to the segment, the field falls off as the inverse distance, but when you get far away, it basically looks like a moving point charge and the field decreases as the inverse square of distance.

Complex circuits involve lots of little current segments, each of which makes its own contribution to the total field. You just superpose all of them at each point in space to get the net field. The resulting field lines are not usually circles or even closed loops. The book describes some particular examples of importance:

- a circular ring of current, from which the field decreases as $1/r^3$ when you get far from it
- a infinite flat sheet of uniform current, for which the field is constant and doesn't get weaker with distance. (You basically can't get away from something infinitely wide!).
- the <u>solenoid</u>, a cylindrical coil of current (like a compressed slinky), inside of which the field lines are all straight and run parallel to each other.

You should familiarize yourself with these examples, as they often come up in practical situations.

The Biot-Savart law

The above descriptions concern the qualitative properties of magnetic fields produced by line currents and individual moving charges. Thinking along those lines should always be your first way of treating this sort of problem. Once you have a feel for what things should look like, you can go ahead and apply formal equations. The general equation is called the Biot-Savart law. Its rawest form involves an integral over a totally general current distribution, i.e., one that can vary arbitrarily.

$$\vec{B} = \frac{\mu_o}{4\pi} \int \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

The current I is along a curve and ds describes the infinitesimal change in length along that curve. The vector version d**s** incorporates a direction, which is tangent to the curve. For each position along the curve, the unit vector **r**-hat is directed from there to the point where **B** is being evaluated, a distance r away. Integration takes you along the curve, so **r**-hat and r change continually, and so does the angle between **r**-hat and d**s**.

As an exercise, consider the field from a moving point charge again, so the integration really includes just a single term. Try to find the direction of **B** by applying the right hand rule for the integrand using its velocity to indicate d**s**. Repeat this for a variety of positions about the particle. You should find that the field direction is exactly the same as found using the thumb-and-curled-fingers method. Another easy example that doesn't require any complicated integration is at the center of a circular loop of current. In that situation, r is constant and d**s** is always perpendicular to **r**-hat, which simplifies the cross product. The answer turns out to be a simple, easily memorizable formula (see text), but once you understand how to derive it, it's just as easy to figure out from scratch using the general Biot-Savart law.

The other important example using the Biot-Savart law is the magnetic field produced by a line current. This derivation requires a more complex treatment, which is covered in the text. There is more elegant way, but that requires a new mathematical tool that represents the same physical information but expresses it in a way that sheds new light on the nature of magnetic fields. That will wait till next time.

*The hard way is to use calculus and integrate. The easy way is to recall what the electric field from an infinite line of charge looked like and make an appropriate analogy. While electric field lines leaving a point charge have two directions in which to spread away from each other, field lines leaving a line charge only have one direction. They can't move away from each other in the direction along the line because there are always other field lines there already. The magnetic situation is more subtle because magnetic field lines don't emanate from the moving charges, but the same idea holds. Whatever magnetic influence is caused by a current, it can't disperse as rapidly in space from a line current as from a single, moving point charge.