Gauss's Law


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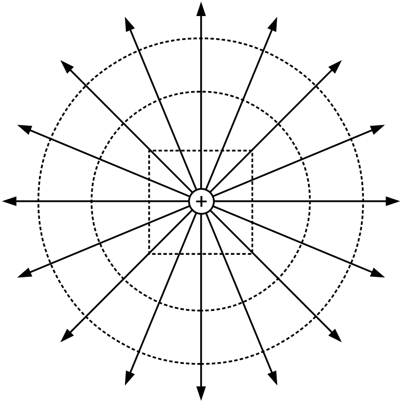
Gauss's Law

Through a *closed* surface,

$$\Phi_E = \frac{Q_{\text{enc}}}{\epsilon_0} = 4\pi k_e\, Q_{\text{enc}}$$

$$\Phi_B = 0$$

$$\Phi_g = -4\pi G\, M_{\text{enc}}$$



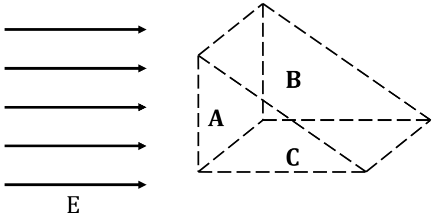
A prism-shaped closed surface is in a constant, uniform electric field E, filling all space, pointing right. The 3 rectangular faces of the prism are labeled A, B, and C. Face A is perpendicular to the electric field. The bottom face C is parallel to the electric field. Face B is the leaning face. (The two triangular side faces are not labeled.) Which face has the largest magnitude of electric flux through it?

A–C. as shown

D. A and B have the same flux

E. They all have zero flux

F. They all have the same non-zero flux



ANS: D—A and B have the same flux

Easy explanation: Flux is a measure of the number of field lines piercing a surface. All field lines shown which pass through A will also pass through B.

More mathematical explanation: Surface B is larger than A, but it does not follow that the flux is larger. Surface B is at an angle to the field. The flux through B is the product of the magnitude of $\vec{E}$ and the component of area B that is perpendicular to $\vec{E}$. It should be easy to see that the component of area B that is perpendicular to $\vec{E}$ is the area A.

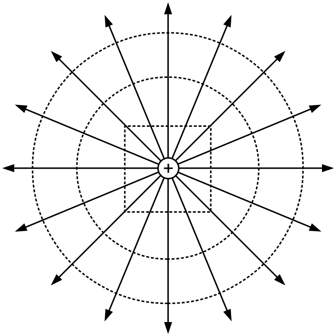
Gauss's law explanation: There is no charge inside the prism. The field is created from charges outside the prism. Therefore, the total flux through all faces is zero.

The flux through face C is zero because the field lines lie within the surface. There is no component of the field perpendicular to the surface C. This same argument shows that the flux through the two triangular faces is also zero.

The above observations lead us to conclude that the total flux through surfaces A and B must be zero. The flux through A is negative because the field lines enter the surface, while the flux through B is positive. The fluxes add to zero, so the fluxes through A and B must have the same magnitude.

Three closed surfaces enclose a point charge. Which surface has the largest flux through it?

A. Small cube
B. Smaller sphere
C. Larger sphere
D. The spheres are equal, but the cube gets more flux
E. Need more info
F. All three have the same flux.



ANS: F—All three have the same flux.

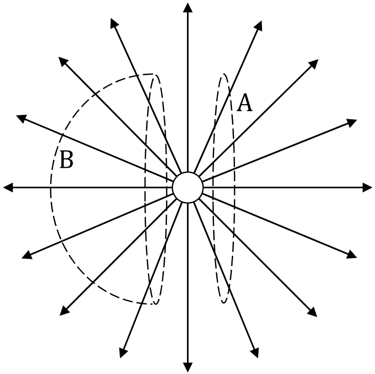
Every electric field line that passes through the small cube also passes through the small sphere, and also through the large sphere. None of the three surfaces has more or fewer field lines passing through it than any other surface. Therefore, they all have the same flux.

Two open surfaces are in an electric field as shown. The right surface A is a flat circular disk of radius R, which squarely faces the charge. The left surface B is a hollow-cup hemisphere of the same radius R. The flat rim of the hemisphere is the same distance from the charge as the rim of the flat disk. Which surface has the greater flux through it?

A–B. as shown

C. They are equal

D. We need to integrate to know for sure.



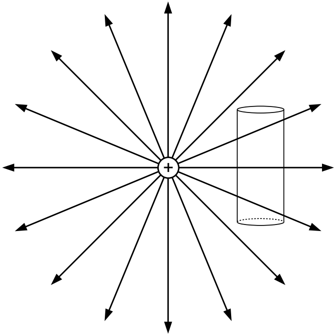
ANS: C—They are equal.

First, note that surface A is identical to the empty "hole" surrounded by the boundary edge of B. If there were a surface in place of this hole, it would have the same flux through it as through A.

Next, note that all of the field lines that pass through the "hole" bound by the edge of B must also pass through surface B. Therefore, B has the same flux through it as does A.

A circular cylinder of radius R and length L is placed near a point charge as shown in the figure. What is the sign of the flux through its *curved* side?

A. Positive (net outward flux)
B. Zero
C. Negative (net inward flux)
D. Need more info



ANS: C—The flux through the curved side is negative.

The flux through the two endcaps is outward (positive). From Gauss's law the total flux through the whole closed surface is zero. Therefore, there must be a net negative flux through the part of the close surface that does not include the endcaps, i.e. the curved side.

You can also see this by noting that some field lines that enter the curved surface on the left to not exit the close surface on the right. Instead, they exit the end caps. Therefore, more field lines enter the curved side surface than exit it, so the net flux through this surface is negative (net inward flux).

A negative point charge with charge $-Q$ sits in the interior of a thick spherical metal shell. The conducting metal shell has zero net charge. What is the total charge on the inner surface of the shell? Hint: consider the indicated Gaussian surface.

A. $-Q$

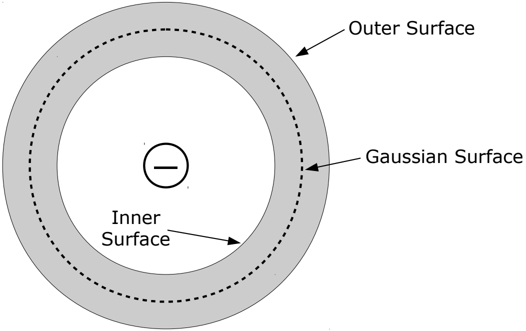
B. $+Q$

C. $-2Q$

D. $+2Q$

E. zero

F. some other answer



ANS: B—The total charge on the inner surface of the shell is $+Q$.

Because the shell is a conductor, we know that $\vec{E} = 0$ inside it. That means the net flux through the spherical Gaussian surface inside the shell is zero.

From Gauss's law this means that the total charge inside the Gaussian surface is zero. The charge at the center is $-Q$, so there must be a charge $+Q$ in the conductor between the inner surface and the Gaussian surface.

Because the argument above can work for a Gaussian surface of any radius as long as it is inside the conductor, this means that the charge $+Q$ must be spread around the inner surface.

Question: Can you determine and explain what the net charge on the outer surface of the conductor must be?

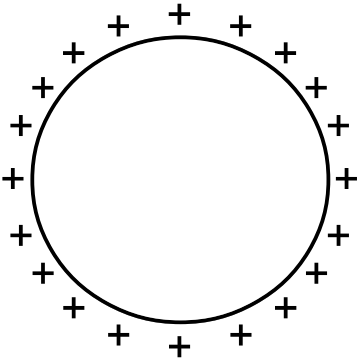
A spherical body has a uniform positive charge density on its surface. There are no other charges around. What can be known about the electric field inside the sphere?

A. $E = 0$ everywhere inside

B. $E = 0$ only at the very center, but E can be non-zero elsewhere inside the sphere.

C. E is non-zero everywhere inside the sphere

D. Not enough info given



ANS: A — $E = 0$ everywhere inside the sphere.

Because the charge is distributed symmetrically over the surface of the sphere, any field inside and outside the object must be spherically symmetric: it must point inward or outward radially, and its magnitude can only depend at most on the distance from the sphere's center.

Place a spherical Gaussian surface of radius r inside the sphere. By spherical symmetry the net flux through this surface must be the field strength multiplied by the surface area, or $\Phi = 4\pi r^2 E$.

Regardless of the Gaussian sphere's radius r, as long as it is less than the radius of the sphere (R), there will be no charge contianed within the surface. Therefore, by Gauss's law, the flux through the Gaussian surface must be zero. This is only possible if the field itself is $E = 0$ for every Gaussian surface with $r < R$. Therefore, $E = 0$ everywhere inside the sphere.

But wait, there's more! Outside the sphere the total charge contained within the Gaussian surface is Q. Therefore, the total flux through the Gaussian surface is $\Phi = 4\pi r^2 E(r) = Q/\epsilon_0$. Solving for E, we get

$$E(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} ,$$

the field for a point charge Q at the origin. Now you know why I told you that the field outside a spherical charge is the same as if all of the charge were located at the center!

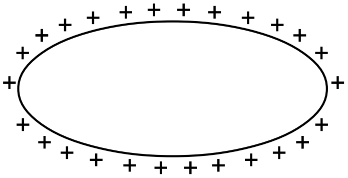
An oblong body has a uniform positive charge density on its surface. There are no other charges around. What can be known about the electric field inside the sphere?

A. $E = 0$ everywhere inside

B. $E = 0$ only at the very center, but E can be non-zero elsewhere inside the sphere.

C. E is non-zero everywhere inside the sphere

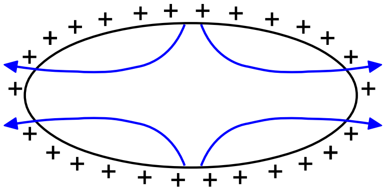
D. Not enough info given



ANS: **B** — $E = 0$ at the center, but can be non-zero elsewhere inside the sphere.

By symmetry it's easy to see that $E = 0$ at the center. At that point the field from charge near the top will be down, while the field from charge near the bottom will be up, and equal in magnitude. The same goes for the field at the center created by charges to the left and right, and other directions. For any given charge location, you can find another charge the same distance from the center in the opposite direction.

However, the charge is not distributed symmetrically over the surface of the sphere, so we cannot expect the field to be exactly zero inside. All we can require is that the net flux through a Gaussian surface inscribed within the body must be zero. This simply means that field lines entering the Gaussian surface must also leave the Gaussian surface. The internal field might look like the diagram below.



Note: For a conductor, the field would be exactly zero inside. This means that the charge must distribute itself on the surface to make the field be exactly zero inside. Such a charge distribution will not be uniform, so we know the object above cannot be a conductor.