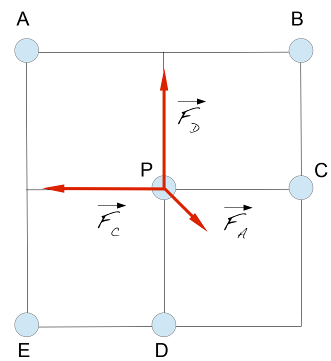
Week 1 Tutorial –Electric Forces – Demonstration Solution

A group of particles are arranged on a square of total side length 32 cm as illustrated below. Particles A-E all carry a positive charge of 15 μC. Particle P at the center has a charge of 75 μC. Find the force on particle P.



Side length L = 32 cm = 0.32 m
For A–E: q = 15 μC = 1.5×10^{-5} C
For P: q_P = 75 μC = $5q$

Strategy: Calculate the individual forces using Coulomb's law and add them as vectors. Because B and E exert equal and opposite forces on P, they cancel & can be neglected.

Consider A, C, D qualitatively first. Because C and D are both closer to P than A is, each of their forces will be larger than A's. The forces on P from C and D are equal and perpendicular, so they will sum to a vector toward A. Adding them as the legs of a right triangle will give an even larger magnitude, which must exceed A's force, while the force from A is away from it. The total force will be up and to the left. Now for the formal math:

Start with A. The distance r_{AP} is half the total diagonal which equals $\sqrt{2}L$. So $r_{AP} = \sqrt{2}L/2 = L/\sqrt{2}$, and the force from A has magnitude

$$F_A = k_e\, q\, 5q/(L/\sqrt{2})^2 = k_e\, 5q^2/(L^2/2^2) = k_e\, 10q^2/L^2$$

That's the magnitude. Note that I used $q_A = 5q$.

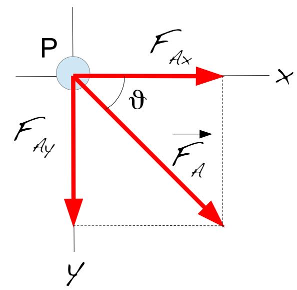
Because $q_C = q$ and $r_{CP} = L/2$, the force from C is to the left and has magnitude

$$F_C = k_e q_C q_P/r_{CP}^2 = k_e 5q^2/(L/2)^2 = k_e 5q^2/(L^2/4) = k_e 20q^2/L^2$$

The force from D has the same magnitude but is pointed upward: $F_D = k_e 20q^2/L^2$

We now have all the magnitudes, so can't we just add them up since we know the total points up and to the left? No!! That would give a number, but a simple sum doesn't take into account that some forces are acting opposite to each other.

Must break up F_A into components! Draw F_A again off to the side for clarity, then drop perpendicular lines to the xy axes to find the components. Identify the sides of right triangles and use trig:



$\cos\theta$ = adjacent/hypotenuse = F_{Ax}/F_A

$\sin\theta$ = opposite/hypotenuse = F_{Ay}/F_A

In this case, $\theta = 45° = \pi/4$, and $\cos\theta = \sin\theta = 1/\sqrt{2}$ (but only for this angle!), so the magnitudes of the components are:

$|F_{Ax}| = F_A\cos\theta = k_e\, 10q^2/L^2 * 1/\sqrt{2}$

$|F_{Ay}| = F_A\sin\theta = k_e\, 10q^2/L^2 * 1/\sqrt{2}$

However, the figure shows that $F_{Ax} > 0$ and $F_{Ay} < 0$, so we'll have to adjust their signs by hand.

The total force is found by adding components. Note that $F_{Cy} = F_{Dx} = 0$ while $F_{Cx} < 0$ and $F_{Dx} > 0$, all by inspection.

$F_x = F_{Ax} + F_{Cx}$

$= k_e\, 10q^2/L^2 * 1/\sqrt{2} - k_e 20q^2/L^2 = -k_e\, q^2/L^2 * \{20 - 10/\sqrt{2}\}$

$F_y = F_{Ay} + F_{Dx}$

$= -k_e\, 10q^2/L^2 * 1/\sqrt{2} + k_e 20q^2/L^2 = k_e\, q^2/L^2 * \{20 - 10/\sqrt{2}\}$

The components of the total force are of equal magnitude, and checking their signs shows it points as we figured out.

Numerical value? Do the leading coefficient first:

$k_e\, q^2/L^2 = (9 \times 10^9\ Nm^2/C^2)\,(1.5 \times 10^{-5} C)^2/(0.32\, m)^2 =$

$\sim (9 \times 2/0.1) \times 10^{9-10}\ N = 200 \times 10^{-1}\ N = 20\ N$

Calc says 19.8 N. Multiple by $\{20 - 10/\sqrt{2}\} \sim = 13$ to get the magnitudes of each component. Calc says 12.9, so

$F_y = 12.9\,(19.8\ N) \sim 13(20 N) = 260\ N = F_x$ Calc says 256 N