MA 231, Guided Notes §1.1

1 Intro

Recall: A *function* is a ________________ that takes a value x as an ________________ and assigns to it a unique value y as an ________________ . Our usual notation for a general function is given by

$$y = f(x),$$

where $f(x)$ is read out loud as "f of x."

Ex: The square function is given as $y = x^2$ or $f(x) = x^2$.

Ex: The sine function is given as ___________________________ .

Ex: A *piecewise function* provides different ________________ depending on the values for x:

$$f(x) =$$

2 Limits

Fundamental to virtually all of calculus is the idea of a *limit*. Though the idea of a limit was known to and used by mathematicians such as ________________

and ________________ as early as the late 1600's, it wasn't until the work of ________________ and ________________ in the early to mid 1800's that our modern day $\epsilon, \delta-$ definition was finally constructed! We leave the formal definition to a later course in real analysis.

<u>**Def:**</u> The **limit** of f as x ________________ c is the value that $f(x)$ gets closer and closer to as x gets __________________________ to c. The notation for this idea is

$$\lim_{x \to c} f(x) = L.$$

It means that as x gets closer and closer to c, $f(x)$ gets closer and closer to L.

Note: The definition expressly forbids letting $x = c$! We are only concerned with what $f(x)$ is doing "near" $x = c$.

Here we introduce a bit of additional notation for limits "from the left" and "from the right." By $\lim_{x \to c^-} f(x)$, we mean the limit of the function f as x approaches c from the left-hand side, i.e. using only values of x where ________________ .

Similarly, By $\lim_{x \to c^+} f(x)$, we mean the limit of the function f as x approaches c from the right-hand side, i.e. using only values of x where ________________ .

In order for an "overall" limit to be equal to L, the left- and right-hand limits have to be the same. I.e. we must have

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L.$$

Example 1: Determine the limit as x approaches 1 of the function $f(x) = x^2$ graphically and numerically.

Example 2: Determine the following limit numerically and graphically:

$$\lim_{x \to 0} \frac{\sin x}{x}$$

Example 3: Given the piecewise function

$$f(x) = \begin{cases} x & x < 1 \\ 2 & x = 1 \\ x & x > 1 \end{cases},$$

determine graphically

$$\lim_{x \to 1} f(x).$$

Example 4: Given the piecewise function

$$f(x) = \begin{cases} x & x < 1 \\ 2 & x = 1 \\ x + 3 & x > 1 \end{cases},$$

determine graphically

$$\lim_{x \to 1} f(x).$$

Example 5: Determine the following limit graphically:

$$\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$$

Example 6: Find $\lim_{x \to 0} 1/x^2$.

3 Conclusion

Ways a limit can fail to exist:

- The function approaches different values as x approaches c from the left and the right.
- The function approaches infinity or negative infinity as x approaches c.
- The function does not settle down to a particular value as x approaches c but instead oscillates infinitely often.