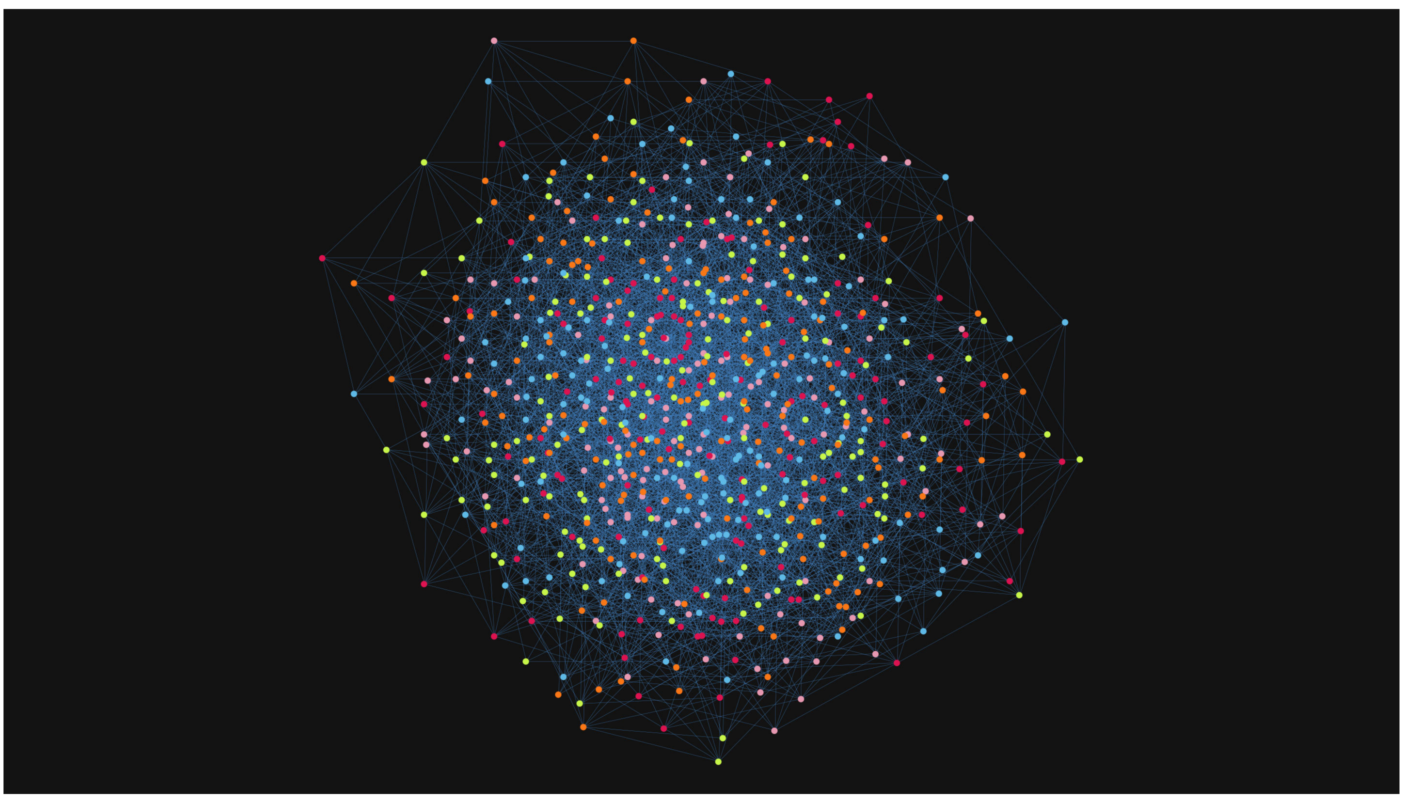
GRAPH THEORY

Decades-Old Graph Problem Yields to Amateur Mathematician

By making the first progress on the "chromatic number of the plane" problem in over 60 years, an anti-aging pundit has achieved mathematical immortality.



This 826-vertex graph requires at least five colors to ensure that no two connected vertices are the same shade. (Click here for a high-resolution version.)

Olena Shmahalo/Quanta Magazine; Source: Marijn Heule

In 1950 Edward Nelson, then a student at the University of Chicago, asked the kind of deceptively simple question that can give mathematicians fits for decades. Imagine, he said, a graph — a collection of points connected by lines. Ensure that all of the lines are exactly the same length, and that everything lies on the plane. Now color all the points, ensuring that no two connected points have the same color. Nelson asked: What is the smallest number of colors that you'd need to color any such graph, even one formed by linking an infinite number of vertices?

The problem, now known as the Hadwiger-Nelson problem or the problem of finding the chromatic number of the plane, has piqued the interest of many mathematicians, including the famously prolific Paul Erdős. Researchers quickly narrowed the possibilities down, finding that the infinite graph can be colored by no fewer than four and no more than seven colors. Other researchers went on to prove a few partial results in the decades that followed, but no one was able to change these bounds.

Then last week, Aubrey de Grey, a biologist known for his claims that people alive today will live to the age of 1,000, posted a paper to the scientific preprint site arxiv.org with the title "The Chromatic Number of the Plane Is at Least 5." In it, he describes the construction of a unit-distance graph that can't be colored with only four colors. The finding represents the first major advance in solving the problem since shortly after it was introduced. "I got extraordinarily lucky," de Grey said. "It's not every day that somebody comes up with the solution to a 60-year-old problem."

Illustration of a 1,581-vertex graph

De Grey’s 1,581-vertex graph. (Click here for a high-resolution version.)

Olena Shmahalo/Quanta Magazine; Source: Aubrey de Grey

The discovery of any graph that requires five colors was a major accomplishment, but mathematicians wanted to see if they could find a smaller graph that would do the same. Perhaps finding a smaller five-color graph — or the smallest possible five-color graph — would give researchers further insight into the Hadwiger-Nelson problem, allowing them to prove that exactly five shades (or six, or seven) are enough to color a graph made from all the points of the plane.

De Grey pitched the problem of finding the minimal five-color graph to Terence Tao, a mathematician at the University of California, Los Angeles, as a potential Polymath problem. Polymath began about 10 years ago when Timothy Gowers, a mathematician at the University of

Cambridge, wanted to find a way to facilitate massive online collaborations in mathematics. Work on Polymath problems is done publicly, and anyone can contribute. Recently, de Grey was involved with a Polymath collaboration that led to significant progress on the twin prime problem.

Tao says not every math problem is a good fit for Polymath, but de Grey's has a few things going for it. The problem is easy to understand and start working on, and there is a clear measure of success: lowering the number of vertices in a non-four-colorable graph. Soon enough, Dustin Mixon, a mathematician at Ohio State University, and his collaborator Boris Alexeev found a graph with 1,577 vertices. On Saturday, Marijn Heule, a computer scientist at the University of Texas, Austin, found one with just 874 vertices. Yesterday he lowered this number to 826 vertices.

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Such work has sparked hope that the six-decade-old Hadwiger-Nelson problem is worth another look. "For a problem like this, the final solution might be some incredibly deep mathematics," said Gordon Royle, a mathematician at the University of Western Australia. "Or it could just be somebody's ingenuity finding a graph that requires many colors."

Correction April 20, 2018: The original version of this article reported that

de Grey had found a "planar unit-distance graph." In graph theory, "planar" means that a graph can be embedded in the plane in such a way that its edges never cross. De Grey's graph is instead a graph in the plane with edges of unit length, or just a "unit-distance graph."

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