Momentum and Impulse

Professor's email to a student:

I was reviewing my records for your recommendation letter, and I show that (your first test grade was) 68%, which is uncharacteristically low for you. I'd just like to confirm that I don't have a typo there. And if 68% is correct, what happened? Just curious.

Student's response:

Indeed! 68% is sadly correct....That test really was a wake up call for me in that class. It made me realize that my standard plug and chug problem solving strategy...wasn't gonna cut it. That's when I came to talk to you during office hours, and I modified my studying strategy to focus more on getting my head around the concepts and how they are all connected...getting more of a full picture understanding of the material being covered rather than how to do specific instances of its application.

Ultimately, this class changed how I study for ALL of my classes nowadays, and I find that the material sticks quite a bit better...it's a heck of a lot more enjoyable as well!

Suppose a ping-pong ball and a bowling ball are rolling toward you. Both have the same momentum, and you exert the same force to stop each. How do the time intervals to stop them compare?

1. It takes less time to stop the ping-pong ball.
2. Both take the same time.
3. It takes more time to stop the ping-pong ball.

ANS: **2**—Both take the same time.

Because the forces are equal, the accelerations will be inversely proportional to the masses. In other words, the lighter ping-pong ball will have a much greater acceleration than will the bowling ball. However, because both ball have the same momentum, the lighter ping-pong ball must also have a much greater initial velocity. The greater acceleration is associated with the necessarily greater change in velocity.

Does this imply that the stopping times are equal? Let's check with numbers. Let m be the mass of the ping pong ball, and $100m$ be the mass of the bowling ball. Both balls have the same momentum, p. The initial speed of the ping-pong ball, therefore, is $v_{\mathrm{p}} = p/m$, while the initial speed of the bowling ball is $v_{\mathrm{b}} = p/(100m) = v_{\mathrm{p}}/100$. Both balls are subject to the same force, F, so the ping-pong ball will have an acceleration of magnitude $a_{\mathrm{p}} = F/m$, while the bowling ball will have acceleration of magnitude $a_{\mathrm{b}} = F/(100m) = a_{\mathrm{p}}/100$. The stopping time for ping-pong ball will be $\Delta t_{\mathrm{p}} = v_{\mathrm{p}}/a_{\mathrm{p}}$, while the acceleration of the bowling ball will be $\Delta t_{\mathrm{b}} = v_{\mathrm{b}}/a_{\mathrm{b}} = (v_{\mathrm{p}}/100)/(a_{\mathrm{p}}/100) = v_{\mathrm{p}}/a_{\mathrm{p}}$. The stopping times are equal.

It is much easier to think of this in terms of force and impulse. Both balls have the same initial and final momenta and therefore require the same impulse to stop them. Impulse is the integral of force over time or, equivalently, the average force multiplied by the time interval. The time required to stop each ball is the change in momentum (impulse) divided by the average force. They have the same momentum change and the same force applied, so it requires the same amount of time to stop each ball.

Suppose a ping-pong ball and a bowling ball are rolling toward you. Both have the same momentum, and you exert the same force to stop each. How do the distances needed to stop them compare?

1. It takes a shorter distance to stop the ping-pong ball.
2. Both take the same distance.
3. It takes a longer distance to stop the ping-pong ball.

A compact car and a large truck collide head on and stick together. Which undergoes the larger momentum change? (Remember, “larger” compares magnitudes.)

1. the car
2. the truck
3. The momentum change is the same for both vehicles.
4. Can’t tell without knowing the final velocity of the combined mass.

ANS: **3**—The momentum change is the same for both vehicles.

Assuming the system is isolated, the momentum of the system is conserved in the collision. Therefore,
$\Delta\vec{\boldsymbol{p}}_{\text{system}} = \Delta\vec{\boldsymbol{p}}_{\text{car}} + \Delta\vec{\boldsymbol{p}}_{\text{truck}} = 0$, so $\Delta\vec{\boldsymbol{p}}_{\text{car}} = -\Delta\vec{\boldsymbol{p}}_{\text{truck}}$.

You can also see this from Newton's Third Law. The car and truck exert equal and opposite forces on each other, for the same amount of time. These are the only forces on the car and truck, therefore the car and truck experience equal impulses and therefore equal changes in momentum.

A compact car and a large truck collide head on and stick together. Which vehicle undergoes the larger acceleration during the collision? (Remember, “larger” compares magnitudes.)

1. the car
2. the truck
3. The acceleration is the same for both vehicles.
4. Can’t tell without knowing the final velocity of combined mass.

ANS: **1**—The car undergoes the larger acceleration.

Both vehicles exert equal forces on each other. There are no other forces on the car, so the collision force is equal to the net force on the car and on the bus. The less-massive car, therefore, will undergo a much greater acceleration.

You can also consider that both cars have the same change in momentum, but the car will have a much greater change in velocity and therefore a much greater acceleration over the time period during which they interact.

The two vectors below describe an object's motion before and after a force is exerted on the object.

$m\vec{\boldsymbol{v}}_i \qquad\qquad m\vec{\boldsymbol{v}}_f = 0$

Which vector illustrates the corresponding impulse?

1. 

2. 

3. 

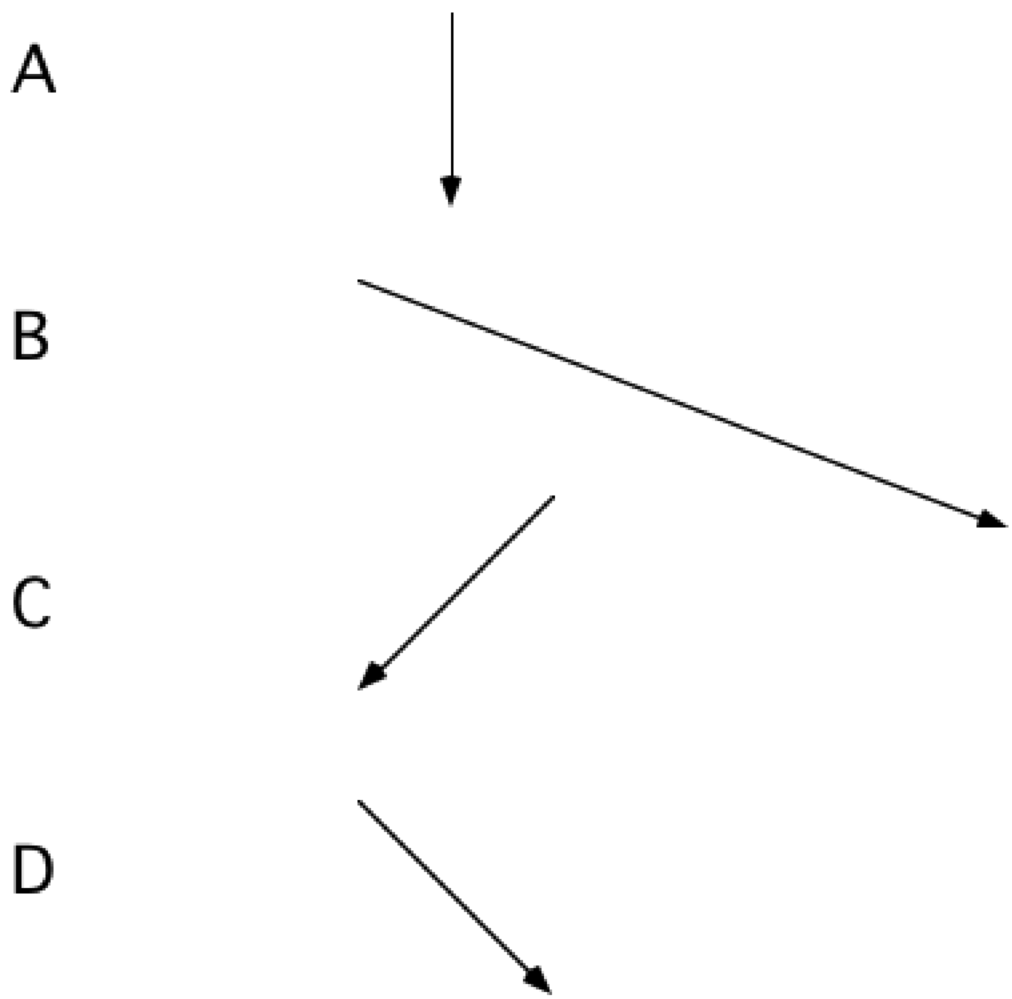
4. 

ANS: The correct answer is **3**.

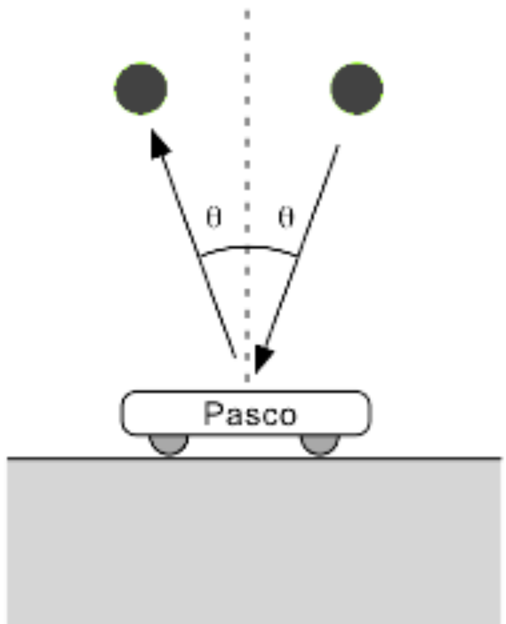
Impulse equals change in momentum. Subtract the initial momentum vector from the final momentum vector. To think about it another way, answer this: what vector, added to the initial momentum, will give the final momentum (in this case, zero)? That vector is the impulse.

The two vectors below describe an object's motion before and after a force is exerted on the object.

Which vector illustrates the corresponding impulse?



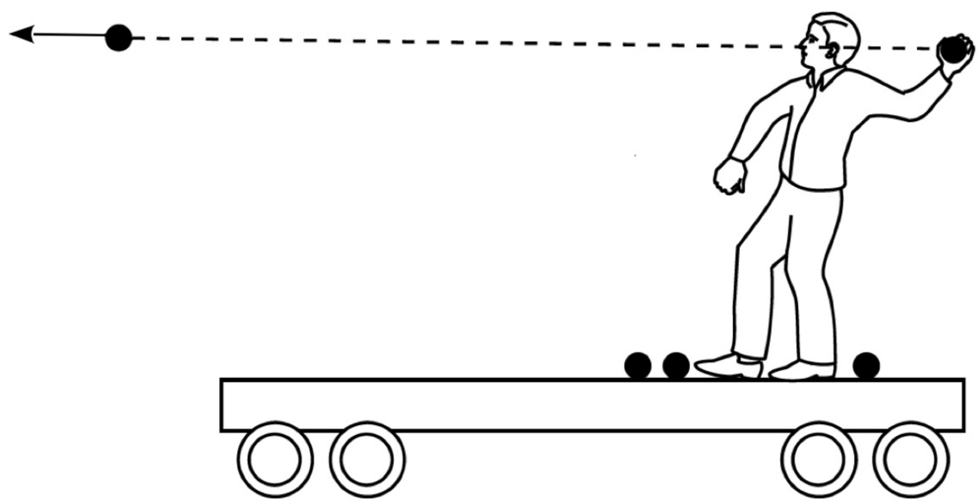
A ball bounces elastically off a cart, as shown below, with no change in speed.



In what direction is the impulse exerted by the ball on the cart?

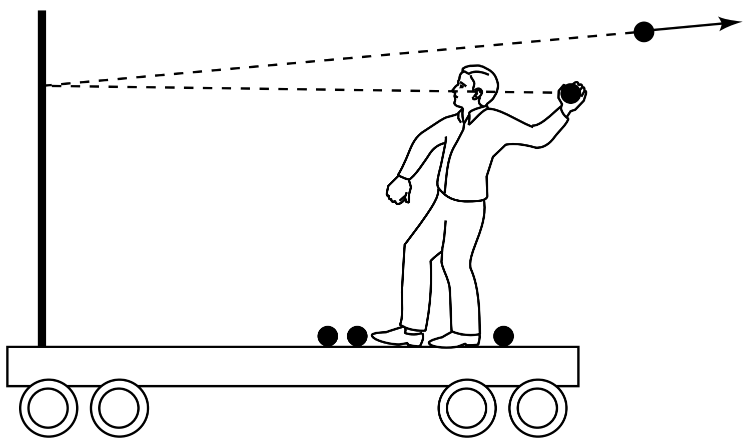
1. Purely downward
2. Purely upward
3. Right and downward
4. Left and downward
5. Other
6. Need more info to determine

Suppose you are on a cart, initially at rest on a track with very little friction. You throw balls straight away from the cart as shown in the figure. Is the cart put in motion?



1. Yes, it moves to the right.
2. Yes, it moves to the left.
3. No, it remains in place.

Suppose you are on a cart, initially at rest on a track with very little friction. You throw balls at a partition that is rigidly mounted on the cart. If the balls bounce straight back as shown in the figure, is the cart put in motion?



1. Yes, it moves to the right.
2. Yes, it moves to the left.
3. No, it remains in place.

ANS: **2**—The cart will move to the left.

Before he throws the ball, the *system* consisting of the cart, person, and balls has zero momentum. After the ball bounces off the wall, that ball has a momentum to the right. That means the cart, person, and the rest of the balls must have an equal momentum to the left. As he continues to bounce balls off the wall, the cart, etc. will continue picking up momentum.

You can also look at this in terms of center of mass. There is no external force on the system consisting of the cart, person, and all of the balls. If one of the balls ends up moving to the right, everything else must end up moving to the left to keep the center of mass of the system fixed at its original position.

Question: What about when he throws the ball? When he throws the ball to the left, he gives it a momentum to the left so the cart and everything else picks up a momentum to the right. Doesn't that cancel the effect?

Answer: It does not. See if you can explain why. (Hint: compare how much rightward momentum the cart gains when he throws the ball, with the leftward momentum it gains when the ball *hits and bounces backward.* What if he were throwing clay balls that stuck to the wall?)

Cars today are designed with crumple zones in front of and behind the passenger compartment, which collapse in a collision. The purpose of these is to

1. reduce the force felt by passengers by decreasing the car's momentum
2. reduce the force felt by passengers by decreasing the impulse they experience
3. reduce the force felt by passengers by increasing the time of impact
4. increase the likelihood you'll have to buy a new car

Warmup Question

Estimate the momentum of a baseball in flight just after the pitcher releases it and compare that to your own momentum at ordinary walking speed.

ANS: A baseball has a mass of around 0.15 kg and a speed around 40 m/s. This gives it a momentum of around 6 kg-m/s. I typically walk on a treadmill at around 1.5 m/s (about 3.3 miles/hr. With a mass of 100 kg my walking momentum would be 150 kg-m/s, many times the momentum of a baseball!

Warmup Question

Your professor has painted the floor and walls of his garage (the one he keeps the massless, frictionless pulleys in) with a special, completely frictionless floor paint. Cross-eyed and dizzy from a long session of untying a jumble of massless strings, he passes out and awakens the next morning in the middle of the floor. How can he get out of this predicament and arrive safely at the garage’s doorway?

1. Take off his shoe and throw it out the open door.
2. Take off his shoe and throw it away from the open door.
3. Both will work.
4. Neither will work.

ANS: **2**—He should take off his shoe and throw it away from the door.

Because the floor is frictionless, he can't walk out (walking is based on static friction between the floor and shoes.) If he throws his shoe away from the door, it will gain a momentum away from the door. From conservation of momentum, he will receive an equal amount of momentum *toward* the door. This momentum will be small, and he may have a very small velocity. However, because the floor is frictionless, there will be no forces keeping him from sliding slowly all the way to the door.