Momentum Conservation

When you identify something you don’t understand, make a note of it so you can direct attention to it. Things you do understand take care of themselves.

A biathlon competitor in the Winter Olympics stands motionless on a slick icy surface and fires her rifle at the target. Which has the higher momentum?

1. The bullet after it leaves the rifle
2. The athlete with the recoiling rifle
3. They are the same
4. The answer depends on the distance over which the bullet accelerates

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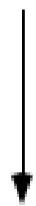
A sailboat carrying a skipper and two mates glides directly toward a dock at speed v_o. Just before it gets there, the second mate at the bow leaps to the dock to tie it off. If that crew member's speed relative to the boat is v_c, then what happens to the boat's speed?

1. It remains unchanged at v_o
2. It is now v_c, but backwards, away from the dock
3. The boat speed is reduced by v_c
4. The boat speed is reduced by less than v_c
5. The boat comes to a halt

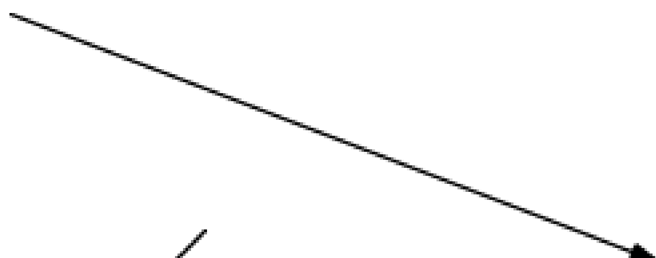
The two vectors below describe an object's motion before and after a force is exerted on the object.

Which vector illustrates the corresponding impulse?

A

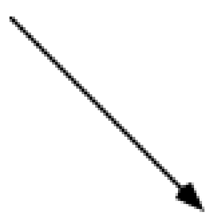


B

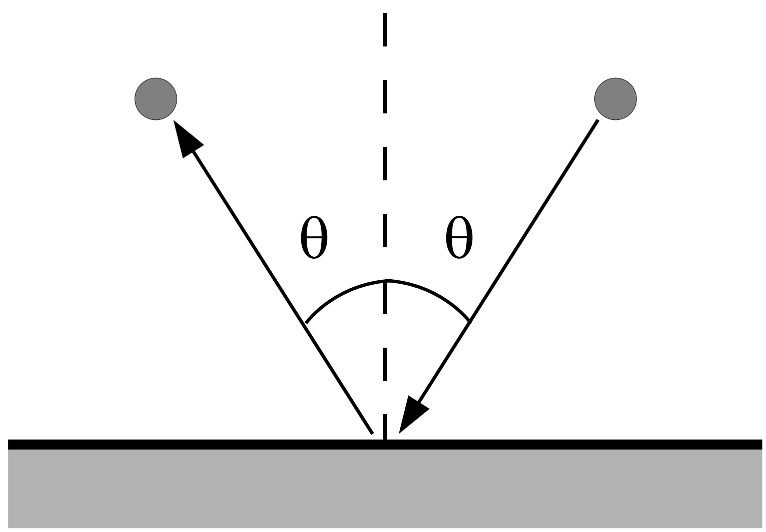


C

D



A ball bounces elastically off a floor, as shown below, with no change in speed. In what direction is the impulse exerted by the ball on the floor?



1. Purely upward
2. Purely downward
3. Down and left
4. Down and right
5. Need more info

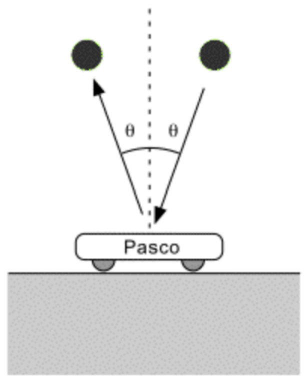
ANS: **2**—The ball exerts a downward impulse on the floor.

The impulse exerted on the floor by the ball will be equal and opposite the impulse exerted by the floor on the ball. Let's consider the impulse on the ball, which is equal to the ball's momentum change. The ball leaves the collision with the same speed it enters with (we call such collisions "elastic.") It also leaves at the same angle relative to the vertical. Therefore, the horizontal component of the ball's velocity after collision will be equal to the horizontal component before collision. This means that the horizontal component of the ball's momentum is constant during the collision, so there is no horizontal impulse on the ball.

The vertical component of the ball's velocity, on the other hand, changes in the collision. The change in the vertical component of velocity, and hence the change in the vertical component of the ball's momentum, is upward. (If we define the upward direction to be positive, the initial vertical component is $v_{iy} = -v\cos\theta$ and the final vertical component is $v_{fy} = +v\cos\theta$, so the change in velocity is $v_{fy} - v_{iy} = +2v\cos\theta$.)

Therefore, the impulse the floor exerts on the ball is purely upward, and the impulse the ball exerts on the floor is purely downward.

A ball bounces elastically off a cart, as shown below, with no change in speed.



In what direction is the force exerted by the ball on the cart?

1. Purely downward
2. Purely upward
3. Right and downward
4. Left and downward
5. Other
6. Need more info to determine

A car accelerates from rest. In doing so the car gains a certain amount of momentum and Earth gains

1. more momentum.
2. the same amount of momentum.
3. less momentum.
4. The answer depends on the interaction between the two.

ANS: **2**—Earth gains the same amount of momentum.

You can think about this in terms of Newton’s third law. The force that accelerates the car forward is static friction between the ground and the tires. This is equal and opposite to the static friction force the tires apply to the ground. Both of these forces operate over the same time interval, so the car and Earth will receive impulses of equal magnitude and opposite direction. Therefore both will have the same amount (magnitude) of momentum change.

Perhaps the better way to think about this is in terms of momentum conservation. Let the car and Earth together be your system, which initially has a total momentum of zero. The friction forces between the car and Earth are internal to the system, so for the purposes of this problem the net external force on the system is zero. Therefore the momentum of the system after the acceleration must still be zero. Whatever forward momentum gained by the car will be balanced by an equal backward momentum gained by Earth.

In the previous question, determine what happens to the center of mass of the Earth-car system. (Treat the Earth-car system as isolated—ignore forces from the sun, moon, stars, galaxies, etc.)

1. It moves in the direction of the car, because the car moves faster than Earth.

2. It moves in the direction of Earth, because Earth is much more massive than the car.

3. It remains in its original position.

ANS: **3**—The center of mass of the Earth-car system remains in its original position.

First, consider the momentum of the car and Earth. As we saw in the previous question, the total momentum of the system is conserved. The car and Earth gain equal and opposite momenta in their interaction. The momentum of the center of mass (CM) of a system is the sum of the momenta of all of its parts, so the total momentum of the system remains zero after the interaction. The total momentum of a system is equal to the total mass of the system multiplied by the velocity of its CM. Therefore, the velocity of the CM is zero, so the CM remains in its original position.

We can also answer this in terms of force and acceleration. The net force on a system is equal to the total mass of the system multiplied by the acceleration of its CM. The Earth-car system is isolated, so there is no net force. Therefore, the center of mass does not accelerate, even though the car and Earth do. If the CM of the system starts at rest, it will remain at rest.

Finally, let's actually compute the position of the CM. Let's assume that the CM is initially at rest at $x_{\mathrm{CM}} = 0$ before the car begins to drive. After the interaction, both the car and Earth will have moved to new points in space, $x_{\mathrm{E}} = \frac{1}{2} a_{\mathrm{E}} \Delta t^2$, and $x_{\mathrm{c}} = \frac{1}{2} a_{\mathrm{c}} \Delta t^2$. Therefore, the location of the CM after time Δt is

$$x_{\mathrm{CM}} = \frac{m_{\mathrm{E}} x_{\mathrm{E}} + m_{\mathrm{c}} x_{\mathrm{c}}}{m_{\mathrm{E}} + m_{\mathrm{c}}} = \frac{\left(m_{\mathrm{E}} a_{\mathrm{E}} + m_{\mathrm{c}} a_{\mathrm{c}}\right) \Delta t^2}{2\left(m_{\mathrm{E}} + m_{\mathrm{c}}\right)} \, .$$

We know from Newton's third law that the force the car exerts on Earth is equal and opposite the force that Earth exerts on the car, and that these forces are equal to the net force on each object. Therefore, $m_{\mathrm{E}} a_{\mathrm{E}} = -m_{\mathrm{c}} a_{\mathrm{c}}$, so $m_{\mathrm{E}} a_{\mathrm{E}} + m_{\mathrm{c}} a_{\mathrm{c}} = 0$. From the equation above, we see that $x_{\mathrm{CM}} = 0$.

A person attempts to knock down a large bowling pin by throwing a ball at it. The ball is covered in velcro (the fuzzy "loop" side), but only one side of the pin has the complementary velcro (the stickery "hook" side). Thus, the ball will either rebound or stick, depending on how the pin is oriented. Which situation is most likely to topple the bowling pin?

1. The ball rebounds directly back off the pin
2. The ball sticks to the pin and clings to it
3. It makes no difference
4. We need more information

Warmup Question

You fire a handgun, a big honkin’ “Dirty Harry” gun. Compare the impulses received by the bullet and by the gun (which you continue to hold).

ANS: Surprisingly, the impulses received by the bullet and by the gun will not be equal. The impulse on the bullet will be due only to the force of the gunshot (gun on the bullet). The force on the gun will be due to the force of the gunshot (bullet on the gun) and the force of your hand holding it. The force your hand exerts on the gun will probably not be as large as the force of the shot, ensuring that the gun will “kick back.” However, this force of your hand on the gun will be directed opposite the force of the bullet on the gun, so the net force on the gun will be less. Therefore, the gun will receive less impulse than the bullet.

If the gun were fired without begin held (say it hung from a string and was fired remotely), both the bullet and the gun will receive equal impulses and therefore equal changes in momenta.

Warmup Question

Estimate the amount of kinetic energy dissipated when two cars of equal mass and moving at highway speeds collide head on.

ANS: Assume a nice round mass for the cars, say 1500kg (about 3000pounds). Highway speeds are $60-70$milesperhour, which converts to a nice round number of 30m/s. The kinetic energy of one car is $\frac{1}{2}mv^2$, so the total energy for two cars with equal mass, m, and speed, v, will be mv^2. Therefore the total initial kinetic energy will be 1.35×10^6J or 1.4MJ.

Since we only want a ballpark estimate, it would be easy to simplify by recognizing that $30^2 = 900 \approx 1000$, which gives a total energy of 1.5MJ. When the cars collide head-on at the same speed, the initial and final momenta should be zero, so both cars will completely stop. That means all of the initial kinetic energy will be lost.

Warmup Question

I once witnessed a car at a stoplight get rear-ended by a large bus. The collision caused the car to shoot quickly forward, while the bus continued to roll forward at a slightly slower speed. Which of the statements below is CANNOT be true?

1. The impulse on the car is greater (in magnitude) than the impulse on the bus.
2. The impulse on the car is equal (in magnitude) to the impulse on the bus.
3. The car gained momentum and kinetic energy in the collision.
4. The bus lost momentum and kinetic energy in the collision.

ANS: **1**—The impulse on the car *cannot* be greater than the impulse on the bus.

There is no net vertical force on the bus or the car, so we can ignore vertical forces when determining impulse, change in momentum, and change in kinetic energy.

The only horizontal force on the car is exerted on the bus, and the only horizontal force on the bus is exerted by the car. As interaction partners, they are equal and opposite, according to Newton's third law. These forces are equal to the net force on each object, so the impulses on the car and bus must be equal in magnitude (and opposite in direction). Therefore, answer 2 is correct (and answer 1 is incorrect).

These impulses are equal to the change in momentum for each object. The car, initially at rest, gains a forward momentum from the forward impulse imparted by the bus. The car's momentum initially was zero, while its final momentum certainly will not be. Therefore, the car will gain momentum in the collision. The car also had no kinetic energy before collision, and certainly will have a kinetic energy after collision. Therefore, the car will gain kinetic energy in the collision. Answer 3 is certainly true.

The backward impulse on the bus is equivalent to a backward change in momentum of the bus. We can look at this change in momentum in two equivalent ways. We can say that the bus will "gain backward momentum," or we can say that it "loses forward momentum." From the description we see that the bus continues to move forward after collision, so it is best to say that it lost forward momentum. Because the bus is moving more slowly than it was before collision, we can also say that it lost kinetic energy.

As long as there are no external forces on the system, the amount of forward momentum lost by the bus will always be equal the amount of forward momentum lost by the bus. However, it is not necessarily true that the amount of kinetic energy lost by the bus will equal the amount of kinetic energy gained by the car. This later statement is only true for elastic collisions.